

Discrete Structures: Love and/or Hate Math?

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Math: Discovered, Invented, or Both?

Sample of answers

- **A canonical answer:** Math is an intricate fusion of inventions and discoveries.
- **A philosopher answer:** Plato argued that mathematics is discovered, not invented. He believed that mathematical concepts, like numbers and shapes, exist independently of the physical world in a realm of perfect, unchanging forms.
- **A physicist answer:** Isaac Newton and Albert Einstein implicitly leaned towards the view that mathematics is a tool invented to describe and understand the natural world, rather than something inherently discovered.
- **A mathematician answer:** “God created the natural numbers; all the rest is the work of man” (Leopold Kronecker).

Online resources

- https://www.youtube.com/watch?v=X_xR5Kes4Rs
- <https://www.youtube.com/watch?v=1i1nEqiGUQM>

“Why Math?” and “What Is Math?”

Who wants to be a millionaire?

- <https://www.youtube.com/watch?v=JpGya7hKtng>
- <https://www.youtube.com/watch?v=BFf4V2YJ1-I>
- <https://www.youtube.com/watch?v=YtrZ4Dec6eo>
- <https://www.youtube.com/watch?v=L7qkKP1iTEA>
- <https://www.youtube.com/watch?v=qkZn2a2EZgs>

Visualize The Problem (The Simpsons)

- https://www.youtube.com/watch?v=6Jq_9ghf-jI

Math as a game!?

- <https://www.youtube.com/watch?v=F92IdQobZiU&feature=youtu.be>

What is Math?

- <https://www.youtube.com/watch?v=64643Op6WJo>

The “map” of Mathematics

- <https://www.youtube.com/watch?v=OmJ-4B-mS-Y&t=425s>

Math and Dates

Find your age: A “magic” mathematical procedure

- Select a number between 1 to 9.
- Multiply by 2.
- Add 5.
- Multiply by 50.
- Add 1775.
- Subtract your birthday year.
- Add 1 if you already had your birthday
 - * Or if it is today – **congratulations!**
- Omit first appearance of selected number if it is in your number.
- You are left with your age!!!

Math and Dates

Find your age: Why it works?

- Manipulating the selected number:

- * Selected number: $d \in \{1, 2, \dots, 9\}$.
- * Age: $k < 100$.
- * After four math operations your number is:

$$n = ((2d + 5) \cdot 50) + 1775 = 2025 + 100d$$

- * Since d is a digit: $n = 2d25$.
 - * $2d25$ becomes 2025 after omitting the digit d .
- Age plus birthday year:
 - * $k + (2025 - k) = 2025$ if birthday after today.
 - * $k + (2026 - k) = 2026$ if birthday before today.

An online resource from 2017

- <https://www.youtube.com/watch?v=VrcW1LbafHo>

Math and Dates

Is 2026 special?

- From a discrete math point of view, perhaps not?
- By contrast, $2025 = 1^3 + 2^3 + \dots + 9^3 = (1 + 2 + \dots + 9)^2$.
- By contrast, 2027 and 2029 are the millennium's first twin primes.

Calculating the day of the Week for any date until 2099

- www.youtube.com/watch?v=UsStBxY9STw&list=PLDZcGqoKA84HZHcTVK2CxXNRsu-2y76ML&index=6&t=0s

The birthday “paradox”

- https://youtu.be/KtT_cgMzHx8

Think Before You Answer!

Can you solve this in five seconds?

- A bat and a ball cost \$11.
- The bat costs \$10 more than the ball.
- How much does the ball cost?

The problem, the solution, and some discussion:

www.youtube.com/watch?v=mjZoLflEhc8&list=PL1-1K8m-elLDitldLRadlsvuCFQVbkgVI&index=80&t=0s

Why so many people give the wrong answer?

<https://www.youtube.com/watch?v=i-aCSFD3xro&feature=youtu.be>

Beyond the Trivial Solution

Problem:

- A and B stand in a line with other people.
- A is the fifth person from right, B is the fifth person from left, and there are three people between them.
- How many people in the line?

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The hidden (?) solution: 5

AXXXB

An online resource

www.youtube.com/watch?v=YuGPE8F0h_4&list=PL1-1k8m-elLDitldLRadlsvuCFQVbkgVI&index=23&t=0s

Is This Really Math?

5 apples to 4 children and 18 horses to 3 children.

- <https://www.youtube.com/watch?v=JixXbESBaLs>
- <https://www.youtube.com/watch?v=mfPAKd3eDwk>

A parking lot sequence with a missing number

- <https://www.youtube.com/watch?v=bwM2-jp3XgA>

Thinking Out of the Box

Covering 9 dots with 4 lines and even with 3 lines

- https://www.youtube.com/watch?v=jvUD_v_f9qQ
- <https://www.youtube.com/watch?v=zs-UtzMstps>

How to connect objects?

- <https://www.youtube.com/watch?v=jrF5ghMvc04>
- <https://www.youtube.com/watch?v=8xRxyfeBUcs>

4 triangles with 6 matches and 7 triangles with 9 matches

- <https://www.youtube.com/watch?v=slyy4RUviNw&feature=youtu.be>
- https://www.youtube.com/watch?v=PtNNqc_oLyI

Two collections each with seven puzzles

- <https://www.youtube.com/watch?v=AFB1q3Bb2pY>
- <https://www.youtube.com/watch?v=B5I1PSxuH5g>

Find the “Shortest” and “Cleverest” Solution!

Problem

One morning a Buddhist monk sets out at sunrise (4am) to climb a path up the mountain to reach the temple at the summit. The monk arrives at the temple just before sunset (7pm). A few days later, the monk leaves the temple at sunrise (4am) to descend the mountain, traveling somewhat faster since it is downhill. Show that there is a spot along the path that the monk will occupy at precisely the same time of the day on both trips.

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Solution

Just visualize Monk 1 going up the hill and Monk 2 coming down the hill. There will be a time in the trip when both of them meet!

An online resource

<https://www.youtube.com/watch?v=WUKVPGd-eJM>

More “Shortest” and “Cleverest” Solutions!

Trains, Birds, and Flies

- <https://www.youtube.com/watch?v=kKMBj1IwU2k>

Two poles and a cable

- https://www.youtube.com/watch?v=1_ffdarcJiQ&feature=youtu.be

A table, a cat, and a turtle

- <https://www.youtube.com/watch?v=BPRueCu3fXU&feature=youtu.be>

Ants on a stick

- Problem: <https://www.youtube.com/watch?v=Hr9grCHHBzw>
- Solution: <https://www.youtube.com/watch?v=KFhZCoU-SlA>

Math and Art

L-System curves

- Walking a plane-filling curve

https://www.youtube.com/watch?v=UKNcbNpP__w

- Levy Dragon L-System Visualization

https://www.youtube.com/watch?v=GVhLhAXAWoI&list=PLZh9gzIvXQUv8taXWI_H1LjjsN2MKcI0c

- Sierpinski Arrowhead Curve L-system

https://www.youtube.com/watch?v=MEzgGGq2l5E&list=PLZh9gzIvXQUv8taXWI_H1LjjsN2MKcI0c

Plotting sequences

- Part I:

<https://www.youtube.com/watch?v=pAMgUB51XZA&list=PLt5AfwLFPxWLkoPqhxvuA8183hh1rBnGM>

- Part II:

<https://www.youtube.com/watch?v=o8c4uYnnNnc>

- Part III:

<https://www.youtube.com/watch?v=j0o-pMIR8uk>

- Part IV:

<https://www.youtube.com/watch?v=4x2kdLFUSvM&list=PLt5AfwLFPxWLkoPqhxvuA8183hh1rBnGM>

Math, Statistics, and Wisdom of Crowds

How many marbles are in the jar?

- <https://www.youtube.com/watch?v=WZtCC-h2pQU&t=174s>

The concept

- <https://www.youtube.com/watch?v=Zv80ibV-0FM>

Do we understand what is “Average”?

The Will Rogers Phenomenon

- “When the Okies left Oklahoma and moved to California, they raised the average intelligence level in both states.”

Example with numbers

- Let $S = \{1, 2, 3\}$ and let $H = \{4, 5, 6, 7\}$.
- The arithmetic averages of both sets are $a_S = 2$ and $a_H = 5.5$.
- Move 4 from H to S so now $S' = \{1, 2, 3, 4\}$ and $H' = \{5, 6, 7\}$.
- The arithmetic averages of S' and H' are $a_{S'} = 2.5$ and $a_{H'} = 6$.
- It follows that $a_{S'} > a_S$ and $a_{H'} > a_H$.

Do we understand what is “Average”?

Manipulating the data

- Professor X taught two sections of Discrete Structures in Brooklyn College.
- 40 students took section I while only 30 students took section II.
- The average final grade in each section was too low.
- Brooklyn college **did not like** this fact!
- Professor X realized that the grades of **all** 40 students in section I were higher than the grades of **all** 30 students in section II.
- Professor X transferred 5 students with the worst grades in section I to section II.
- Professor X resubmitted the new average grades of both sections.
- Brooklyn College **liked** the new report better!

Do we understand what is “Average”?

A “dangerous” example

- Let S be the set of all sick people in the world and let a_S be their life expectancy. Let H be the set of all healthy people in the world and let a_H be their life expectancy.
- New diagnostic methods move people from H to S . Let M be the set of all people in the world who moved from H to S and let a_M be their life expectancy. Let $S' = S \cup M$ and let $H' = H \setminus M$.
- Naturally, $a_M > a_S$ and $a_M < a_H$. As a result, $a_{S'} > a_S$ and $a_{H'} > a_H$.
- But the life expectancy of people in the world remains the same! New diagnostic methods by themselves do not influence life expectancy!

An online resource

- <https://www.youtube.com/watch?v=J4NtCsrwq2E>