Solutions to Discrete Structures TR11 Number Bases Quiz

1. Write the decimal number $(40)_{10}$ in its binary (base-2) and its ternary (base-3) representations.

$$\begin{aligned} \mathbf{40} = 32 + 8 = 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 &\implies (40)_{10} = (\mathbf{101000})_{\mathbf{2}} \\ \mathbf{40} = 27 + 9 + 3 + 1 = 1 \cdot 3^3 + 1 \cdot 3^2 + 1 \cdot 3^1 + 1 \cdot 3^0 &\implies (40)_{10} = (\mathbf{1111})_{\mathbf{3}} \end{aligned}$$

2. Write the twelve decimal numbers $(1)_{10}, (2)_{10}, \ldots, (12)_{10}$ in their base-6 representation.

$$\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5}, \mathbf{10}, \mathbf{11}, \mathbf{12}, \mathbf{13}, \mathbf{14}, \mathbf{15}, \mathbf{20}$$

3. Which number is larger $(212)_4$ or $(100110)_2$?

$$\begin{aligned} (\mathbf{212})_{\mathbf{4}} &= 2 \cdot 4^2 + 1 \cdot 4^1 + 2 \cdot 4^0 = 32 + 4 + 2 = \mathbf{38} \\ (\mathbf{100110})_{\mathbf{2}} &= 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 32 + 4 + 2 = \mathbf{38} \end{aligned}$$

4. In the base-14 system: $A = 10$, $B = 11$, $C = 12$ and $D = 13$. What is the decimal value of the number $(BAD)_{14}$?

$$(\mathbf{BAD})_{\mathbf{14}} = 11 \cdot 14^2 + 10 \cdot 14^1 + 13 \cdot 14^0 = 2156 + 140 + 13 = (\mathbf{2309})_{\mathbf{10}}$$

5. Characterize **all** base-3 numbers that are not multiples of 3 (i.e., those not divisible by 3).

   Answer: A base-3 number x is a multiple of 3 if and only if it ends in 0. Thus, any base-3 number ending in 1 or 2 is not divisible by 3.

   Generalization: A base-b number $x = (d_k d_{k-1} \ldots d_0)_b$ is a multiple of b if and only if $d_0 = 0$.

   Explanation: By definition, $x = \sum_{i=0}^{k} d_i b^i$. Since the terms $d_i b^i$ for $i \geq 1$ are all multiples of b, it follows that x is divisible by b if and only if the constant term d_0 (the last digit) is zero.

6. What is the effect of appending 1 to the end of a base-7 number x?

   Answer: $(\mathbf{x1})_{\mathbf{7}} = \mathbf{7x + 1}$.

   Generalization: What is the effect of appending a digit c (where $0 \leq c < b$) to the end of a base-b number x?

   Answer: $(\mathbf{xc})_{\mathbf{b}} = \mathbf{bx + c}$.

   Explanation: Appending a zero to the end of a base-b number is equivalent to shifting all digits one position to the left, which scales the value by a factor of b. Replacing that trailing zero with the digit c is equivalent to adding c to the total value. Therefore, the resulting value is $bx + c$.