Solutions to Discrete Structures TR2 Number Systems Quiz

1. Write the decimal number $(63)_{10}$ in its binary (base-2) and its ternary (base-3) representations.

$$\mathbf{63} = 32 + 16 + 8 + 4 + 2 + 1 = 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 \implies (63)_{10} = (\mathbf{111111})_\mathbf{2}$$
$$\mathbf{63} = 2 \cdot 27 + 9 = 2 \cdot 3^3 + 1 \cdot 3^2 + 0 \cdot 3^1 + 0 \cdot 3^0 \implies (63)_{10} = (\mathbf{2100})_\mathbf{3}$$

2. Write the twelve decimal numbers $(1)_{10}, (2)_{10}, \ldots, (12)_{10}$ in their base-5 representation.

$$\mathbf{1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, 22}$$

3. Which number is larger $(222)_4$ or $(101010)_2$?

$$\begin{aligned} (\mathbf{222})_\mathbf{4} &= 2 \cdot 4^2 + 2 \cdot 4^1 + 2 \cdot 4^0 = 32 + 8 + 2 = \mathbf{42} \\ (\mathbf{101010})_\mathbf{2} &= 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 32 + 8 + 2 = \mathbf{42} \end{aligned}$$

4. In the base-15 system: $A = 10$, $B = 11$, $C = 12$, $D = 13$, and $E = 14$. What is the decimal value of the number $(ACE)_{15}$?

$$(\mathbf{ACE})_\mathbf{15} = 10 \cdot 15^2 + 12 \cdot 15^1 + 14 \cdot 15^0 = 2250 + 180 + 14 = (\mathbf{2444})_\mathbf{10}$$

5. Characterize **all** base-4 numbers that are multiples of 2 (i.e., those divisible by 2)?

   Answer: A base-4 number x is a multiple of 2 if and only if its least significant digit is even (i.e., it ends in 0 or 2).

   Generalization: For any even base b, a base-b number x is a multiple of 2 if and only if its least significant digit is even (i.e., it ends in $0, 2, \ldots$).

   Explanation: By definition, $x = \sum_{i=0}^{k} d_i b^i$. Since the terms $d_i b^i$ for $i \geq 1$ are all multiples of b, it follows that for an even b these terms are multiples of 2. Consequently, the parity of x is determined solely by the parity of its constant term d_0 (the last digit). As a resultm x is divisible by 2 if and only if the last digit is even.

6. What is the effect of appending 3 to the end of a base-5 number x?

   Answer: $(\mathbf{x3})_\mathbf{5} = \mathbf{5x + 3}$.

   Generalization: What is the effect of appending a digit c (where $0 \leq c < b$) to the end of a base-b number x?

   Answer: $(\mathbf{xc})_\mathbf{b} = \mathbf{bx + c}$.

   Explanation: Appending a zero to the end of a base-b number is equivalent to shifting all digits one position to the left, which scales the value by a factor of b. Replacing that trailing zero with the digit c is equivalent to adding c to the total value. Therefore, the resulting value is $bx + c$.