2210 Discrete Structures Expanded Prerequisite Quiz: **Solutions**

1. The five components of the famous formula $e^{\pi i} + 1 = 0$ are:
   (a) The additive identity: $\underline{\mathbf{0}}$
   (b) The multiplicative identity: $\underline{\mathbf{1}}$
   (c) The ratio of a circle's circumference to its diameter: $\underline{\pi}$
   (d) The base of the natural logarithm: $\underline{\mathbf{e}}$
   (e) The square root of -1: $\underline{\mathbf{i}}$

2. $\underline{\mathbf{0}} < \underline{\mathbf{1}} < \underline{\mathbf{\sqrt{2} = 1.414\ldots}} < \underline{\mathbf{e = 2.718\ldots}} < \underline{\mathbf{\pi = 3.141\ldots}}$

3. (a) $\mathbf{3+3\times 3+3 = 3+(3\times 3)+3 = 3+9+3 = 15}$
   (b) $\mathbf{3\times 3+3\times 3 = (3\times 3)+(3\times 3) = 9+9 = 18}$
   (c) $\mathbf{4\div 4\div 4 = (4\div 4)\div 4 = 1\div 4 = 1/4}$

4. (a) i. $\mathbf{(x+y)^2 = x^2+2xy+y^2}$
   ii. $\mathbf{(x-y)^2 = x^2-2xy+y^2}$
   iii. $\mathbf{(x+y)^3 = x^3+3x^2y+3xy^2+y^3}$
   iv. $\mathbf{(x-y)^3 = x^3-3x^2y+3xy^2-y^3}$
   v. $\mathbf{(x+y)^n = \sum_{i=0}^{n}\binom{n}{i}x^{n-i}y^i = x^n+nx^{n-1}y+\cdots+nxy^{n-1}+y^n}$
   vi. $\mathbf{(x-y)^n = \sum_{i=0}^{n}(-1)^i\binom{n}{i}x^{n-i}y^i = x^n-nx^{n-1}y+\cdots\pm nxy^{n-1}\pm y^n}$

   (b) i. $\mathbf{x^2-y^2 = (x-y)(x+y)}$
   ii. $\mathbf{x^3-y^3 = (x-y)(x^2+xy+y^2)}$
   iii. $\mathbf{x^3+y^3 = (x+y)(x^2-xy+y^2)}$
   iv. $\mathbf{x^n-y^n = (x-y)(x^{n-1}+x^{n-2}y+x^{n-3}y^2+\cdots+x^2y^{n-3}+xy^{n-2}+y^{n-1})}$
   v. $\mathbf{x^n+y^n = (x+y)(x^{n-1}-x^{n-2}y+x^{n-3}y^2-\cdots+x^2y^{n-3}-xy^{n-2}+y^{n-1})}$
   only for odd n.

5. (a) i. $\mathbf{x^n\times x^m = x^{n+m}}$
   ii. $\mathbf{x^n\times y^n = (xy)^n}$
   iii. $\mathbf{\frac{\log_a(x^n)}{\log_a(x)} = n}$
   iv. $\mathbf{2\log_a\left(\sqrt{x}\right) = \log_a(x)}$

   (b) i. If $\log_a(y) = x$, then $\mathbf{a^x = y}$
   ii. If $\log_a(x)+\log_a(y) = \log_a(z)$, then $\mathbf{z = xy}$
   iii. If $\log_a(x)-\log_a(y) = \log_a(z)$, then $\mathbf{z = x/y}$

6. (a) $\mathbf{4! = 4 \times 3 \times 2 \times 1 = 24}$

   (b) $\mathbf{5! = 5 \times 4 \times 3 \times 2 \times 1 = 120}$

   (c) $\mathbf{6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720}$

   (d) $\mathbf{\frac{(n+1)!}{(n-1)!} = \frac{(n+1)\times n\times(n-1)\times\cdots\times 2\times 1}{(n-1)\times(n-2)\times\cdots\times 2\times 1} = (n+1)n}$

7. (a) For the following 2 linear equations

$$\begin{aligned} x + y &= a \\ bx + cy &= 0 \end{aligned}$$

   The values of x and y as a function of the three constants a, b, c are:

$$\mathbf{x = \frac{ac}{c-b}} \quad \text{and} \quad \mathbf{y = \frac{-ab}{c-b}}$$

   (b) The roots of the quadratic equation $x^2 + bx = c$ (equivalently, $x^2 + bx - c = 0$) are

$$\mathbf{x_1 = \frac{-b + \sqrt{b^2 + 4c}}{2}} \quad \text{and} \quad \mathbf{x_2 = \frac{-b - \sqrt{b^2 + 4c}}{2}}$$

8. Four fair coins are flipped. The 16 possible outcomes are

- **HHHH**, **HHHT**, **HHTH**, **HHTT**, **HTHH**, **HTHT**, **HTTH**, **HTTT**, **THHH**, **THHT**, **THTH**, **THTT**, **TTHH**, **TTHT**, **TTTH**, **TTTT**

   (a) The probability that all the coins show H is **1/16**

   - **HHHH**

   (b) The probability that exactly one coin shows H while the other three coins show T is $\mathbf{4/16 = 1/4}$

   - **HTTT**, **THTT**, **TTHT**, and **TTTH**

   (c) The probability that exactly two coins show T and two coins show H is $\mathbf{6/16 = 3/8}$

   - **TTHH**, **THTH**, **THHT**, **HTTH**, **HTHT**, and **HHTT**

9. (a) i. Let T be a right-angled triangle with sides a, b, and c where c is the hypotenuse (the side opposite the right angle). Then

$$\mathbf{c^2 = a^2 + b^2 \implies c = \sqrt{a^2 + b^2}}$$

ii. Let T be a triangle in which one of its side is of length b. Let h be the length of the height that is perpendicular to the side b. Then the area of the triangle T is

$$\mathbf{\frac{b \cdot h}{2}}$$

(b) The sum of the degrees of all the inner angles of the following geometric shapes is

i. Triangle: **180**
ii. Square: **360**
iii. Pentagon: **540**
iv. Hexagon: **720**
v. n-gon: $\mathbf{180(n-2)}$

(c) Let C be a circle whose radius is r and whose diameter is d.

i. The circumference of C as a function of r is $\mathbf{2\pi r}$
ii. The circumference of C as a function of d is $\mathbf{\pi d}$
iii. The area of C as a function of r is $\mathbf{\pi r^2}$
iv. The area of C as a function of d is $\mathbf{\frac{\pi d^2}{4}}$

10. (a) $f(n)$ (* $n \geq 1$ is an integer *)
$c = 0$
for $i = 1$ **to** n **do**
for $j = 1$ **to** n **do**
$c := c + 1$
$\implies \mathbf{c = n^2}$

(b) $f(n)$ (* $n \geq 1$ is an integer *)
$c = 1$
for $i = 1$ **to** n **do**
$c := c * 2$
$\implies \mathbf{c = 2^n}$

(c) $f(n)$ (* $n \geq 1$ is a power of 2 integer *)
$c = 0$
while $n > 1$ **do**
$n := n/2$
$c := c + 1$
$\implies \mathbf{c = \log_2(n)}$

11. The following functions are ordered (left to right) by their growth from the slowest to the fastest when n tends to infinity:

$$\underline{\mathbf{\log(n) \; < \; \sqrt{n} \; < \; n \; < \; n^2 \; < \; 2^n \; < \; n^n}}$$