Non-Monotonic Inference on Sequenced Belief Bases (Preliminary Report)

Samir Chopra
Department of Philosophy
CUNY Graduate Center
schopra@csp.gc.cuny.edu

Konstantinos Georgatos
Department of Computer Science
Brooklyn College of CUNY
kostas@csp.gc.cuny.edu

Rohit Parikh
Departments of Computer Science, Mathematics and Philosophy
Brooklyn College and CUNY Graduate Center
ripbc@cunyvm.cuny.edu

June 4, 1999

1 Introduction

Belief revision is the process of transforming a belief set upon receipt of new information. Given a theory K and a proposition α, [AGM85] propose postulates for $K * \alpha$, the revised theory with α. But AGM-like postulates do not specify how we *came* to believe K and after revision, it is assumed that (K and) $K * \alpha$ is a generic theory. But in practice it is something more, we know that α was our *last* information. We suggest that $K * \alpha$ is not a theory, but the (sequenced) belief *base* $K; \alpha$ i.e. a sequence of propositions with α being the most recent. This observation that the most perspicuous way to represent our beliefs is a belief base has been noted by [Han92] and [Neb92]. Similarly, the notion that a sequence of formulae captures the importance of temporal ordering is noted by [Leh95].

We propose a method for inference from sequenced belief bases. This method augments recent proposals, [Geo96], [PAR96] and [CP99]. In [Geo96], it is shown that taking the linear order of a belief sequence as a prioritization generates a variety of inference relations. It is shown that all such schemes are non-monotonic (rational inference) and, therefore, induce a method for belief revision. In [PAR96] it is shown that if we have a theory referring to two disjoint subjects, then our language can be partitioned into corresponding sub-languages, and it is suggested that new information about one of them should not affect the other. This ensures a relevance or context sensitive, *localized* notion of belief revision. [CP99] explicate the distinction between implicit and explicit beliefs by considering sets of theories called B-structures, which are individually consistent, but can be *jointly* inconsistent. This captures the intuition that real agents often reason with an inconsistent, yet usable, set of beliefs.

In a similar spirit, our (current) method of inference blocks the derivation of explicitly inconsistent beliefs from a possibly inconsistent belief base by using a notion of inference from *maxiconsistent subsets of relevant formulas*. Choosing maxiconsistent subsequences in order to avoid inconsistency was proposed in [Geo96], while relevance is determined, as in [PAR96] and [CP99] by language overlap or by other context determined features. The formula whose inference from the base is to be determined imposes a prioritization on the formulas in the base (thus reorganizing the temporal ordering present in the base). Therefore, we do not treat a belief base as a set but rather as a linear order much like an entrenchment (see [GM88], [Geo97]). Since we take relevance into account we cannot use an entrenchment ordering in our method. This procedure for inference serves as a generalization of the method presented in [Geo96] and [CP99]. In this way, we hope to present a model for belief revision that is a plausible representation of real agents' reasoning.

Notation: In the following, $\mathcal{L}$ is a finite propositional language with the usual logical connectives ($\neg, \vee, \wedge, \rightarrow, \leftrightarrow$). The constants *true, false* are in $\mathcal{L}$. Greek letters α, β, γ denote arbitrary formulae while Roman lower case letters p, q, r denote propositional atoms. $\alpha \Leftrightarrow \beta$ means that $\alpha \leftrightarrow \beta$ is a tautology. We reserve the letters σ, τ for sequenced belief bases. For conciseness we will often refer to sequenced belief bases as belief bases or sequences.

2 Sequenced Belief Bases

We begin with a few preliminary definitions.

Definition 1 *The* language L_α of α *is the smallest set of propositional variables used to express α.*

So, if $\alpha = p_1 \rightarrow (p_2 \vee \neg p_2)$ then $L_\alpha = \{p_1\}$. (L_α is unique, cf. Lemma LS1 in [PAR96])

Definition 2 *Two formulae $\beta_1, \beta_2 \in \mathcal{L}$ are* disjoint *iff their languages are disjoint, i.e. iff $L_\alpha \cap L_\beta = \emptyset$.*

If two disjoint formulae are individually consistent, then they are jointly consistent.

Definition 3 *A sequenced belief base is a sequence of formulae under a temporal ordering, i.e. a sequence of formulae, $\sigma = \beta_1 \dots \beta_n$ where for any pair of beliefs β_i, β_j if $i < j$, β_j is more recent than β_i. Given two sequences, σ_1, σ_2 we say that $\sigma_1 \sqsubseteq \sigma_2$ if σ_2 is obtained from σ_1 by the concatenation of one or more formulas; σ_1 will be referred to as an initial segment of σ_2*

Thinking of our belief base as a sequence of formulas lets us draw upon the intuitions expressed in [Leh95]. Under a temporal ordering the most recent formulae occur at the tail of the sequence. We assume that each formula in the sequence is expressed in its smallest language as above.

We now give a context-sensitive measure for *relevance* amongst formulas in a belief base.

Definition 4 *A pair of formulas, α, β are directly relevant if they are not disjoint i.e. if $L_\alpha \cap L_\beta \neq \emptyset$. Given a sequenced belief base σ, a pair of formulas α, β are k-relevant wrt σ if $\exists \chi_1, \chi_2, \dots \chi_k \in \sigma$ such that:*
i) α, χ_1 are directly relevant
ii) χ_i, χ_{i+1} are directly relevant for $i = 1, \dots k-1$
iii) χ_k, β are directly relevant. (NB: the base σ will be omitted when clear from the context). If $k = 0$ above, the formulas are directly relevant. A pair of formulas are irrelevant if they are not relevant. Let $rel(\alpha, \beta, \sigma)$ be the lowest k such that α, β are k-relevant wrt σ.

Remark:

1. If a pair of formulas are k-relevant, then, $\forall m > k$, they are k-relevant as well.

2. Let $Set(\sigma) = \{\alpha \mid \alpha \text{ occurs in } \sigma\}$. Then, $\sigma_1 \subseteq' \sigma_2$ iff $Set(\sigma_1) \subseteq Set(\sigma_2)$. Note then, that if $\sigma_1 \subseteq' \sigma_2$, then $rel(\alpha, \beta, \sigma_2) \leq rel(\alpha, \beta, \sigma_1)$.

The above definition extends the definition of relevance used in [PAR96] and makes explicit the contextual nature of the relevance definition: two formulas α, β may have different degrees of relevance with respect to different belief bases. A belief base defines a particular context or set of subject matters; pairs of formulas acquire different relationships to one another given differing contexts.

3 Revision and Inference on Sequenced Belief Bases

3.1 Revision and Expansion

Revision and expansion on sequenced belief bases is easy to achieve. In the case of revision, we simply concatenate the new formula γ to the sequence. It becomes the most recent piece of information received. That is, the sequence $\beta_1, \ldots, \beta_n$ simply acquires a new member β_{n+1} added on at the end of the sequence.

3.1.1 Prioritized Inference

We use the *maxiconsistent* approach to define *prioritized* inference on a belief base σ. This method employs a restricted part of σ: a maxiconsistent subset. (σ itself can be an inconsistent set). The elements of a sequenced belief base σ are reorganized by the context created in answering the query 'γ?' (γ is a formula whose deducibility from that base is being determined). That is, answering a query causes a particular structuring or reshuffling of the agent's sequenced belief base; the temporal ordering on the formulae in the base is reorganized into a relevance ordering. We describe this method for answering queries as follows: consider a formula γ in its smallest language L_γ. We construct a maxiconsistent subset $\Gamma_{\langle\sigma,k,\gamma\rangle}$ (of k-relevant to γ formulas) of σ. The construction of this set is regulated by the ordering $\prec$ that γ creates on σ.

Definition 5 *$\beta_i \prec \beta_j$ if a) $\exists r$, β_i is r-relevant to γ and β_j is not r-relevant to γ (i.e. β_i is more relevant to γ than β_j) or b) β_i, β_j are equally relevant but $j < i$, i.e. β_i is more recent than β_j.*

The $\delta_1, \ldots, \delta_n$ are the $\beta_1, \ldots, \beta_n$ under this order. In the definition below we drop the subscripts to improve readability i.e Γ is the set $\Gamma_{\langle\sigma,k,\gamma\rangle}$ referrred to above.

Definition 6 $\Gamma^0 = \emptyset$, $\Gamma^{i+1} = \begin{cases} \Gamma^i & \textit{if } \Gamma^i \vdash \neg\delta_{i+1} \textit{ or if } \delta_{i+1} \textit{ is not } k\textit{-relevant to } \gamma \\ \Gamma^i \cup \{\delta_{i+1}\} & \textit{otherwise} \end{cases}$
$\Gamma_{\langle\sigma,k,\gamma\rangle} = \bigcup_{i=1}^{n} \Gamma^i_{\langle\sigma,k,\gamma\rangle} = \Gamma^n$.

We check formulas for addition to $\Gamma_{\langle\sigma,k,\gamma\rangle}$ in order of their decreasing relevance (increasing k-relevance) to γ. The lower the level of relevance allowed (i.e. the higher the value of k), the larger the part of σ considered. We now define the inference operation $\vdash^k$.

Definition 7 *$\sigma \vdash^k \gamma$ iff $\Gamma_{\langle\sigma,k,\gamma\rangle} \vdash \gamma$*

Once $\Gamma_{\langle\sigma,k,\gamma\rangle}$ has been constructed, can answer queries with definite responses.

Definition 8 *If* $\Gamma_{\langle\sigma,k,\gamma\rangle} \vdash \gamma$*, then answer 'yes'.*
If $\Gamma_{\langle\sigma,k,\gamma\rangle} \vdash \neg\gamma$ *then answer 'no'.*
Otherwise, answer 'no information'.

Notice that even if the sequence σ is inconsistent, the agent is able to give consistent answers to every query. Our inference method corresponds to the *liberal* inference defined on a linearly prioritized sequence of formulas in [Geo96]. [Geo96] defines a *strict* notion of inference as well, which in our case would correspond to stopping the construction of $\Gamma_{\langle\sigma,k,\gamma\rangle}$ upon encountering the first formula that would make the set inconsistent.

3.2 Properties of $\vdash^k$

Proposition 1 *The inference procedure defined above is monotonic in* k*, the degree of relevance i.e. if* $\sigma \vdash^k \gamma$ *then* $\sigma \vdash^{k+1} \gamma$*.*

The above follows immediately from the definition of the inference procedure. There is no loss of information then, in stopping at a particular value of k since any formula derivable at that point will be derivable later as well. This property has the value of being sensitive to the resources available to us during a particular inference operation since we can choose to stop as soon as we get an answer.

Proposition 2 *The inference procedure is non-monotonic in expansions of a sequenced belief base i.e. if* $\sigma \sqsubseteq \sigma'$ *and* $\sigma \vdash^k \gamma$ *then it is not necessarily the case that* $\sigma' \vdash^k \gamma$*.*

The above is obvious by the definition of the inference procedure.

Remarks:

1. In [CP99] the authors presented a method for answering queries that allowed the answer $\top$, i.e. (*over-defined* or *inconsistent*). In the current paper we block the possible inconsistency of Γ^n, thus preventing an inconsistent answer. The procedure of [CP99] will agree with our method when the former gave 'yes' or 'no' answers.

2. The B-structure query answering method corresponds to $\vdash^0$ i.e. for Γ^0 constructed only considering directly relevant formulas.

3. Given the notion of inference defined above, we say that belief bases which yield the same answers to all queries are equivalent. The task of reducing bases to their simplest equivalent form is, however, computationally non-trivial.

4. It is natural to ask how our inference compares with AGM-like postulates. Unfortunately we cannot apply the results in [Geo96] that show that maxiconsistent inference on a belief sequence is rational. The reason is that we take into account relevance that cause a reordering of our sequence depending on the consequence we are testing. However, there are some AGM properties that do hold, like the following:

   $AGM'3$: If $\vdash \alpha \leftrightarrow \beta$ then $\forall\gamma\ \alpha \vdash^k$ iff $\sigma * \beta \vdash^k \gamma$

   Since α, β are expressed in their smallest language, this result is immediate.

4 Conclusion

The process of revision we described is easy to implement as we only need to keep track of the base. We do not drop any incoming information; it might be useful in later stages of revision. In the case of sequences of formulas, the order of the incoming information plays a role in the inference procedures. Often, newer information renders previous beliefs obsolete. Our method extends these intuitions as well as incorporating those mentioned previously. Such an approach is plausible since in real life we are unable to keep track of all available information and must rely on notions like relevance and context to aid us. Our procedure presented above provides a plausible, tractable method for belief representation and answering queries. In future work, we plan to implement this method and further investigate its properties.

References

[AGM85] C. E. Alchourrón, Peter Gärdenfors, and David Makinson. On the logic of theory change: partial meet contraction and revision functions. *The Journal of Symbolic Logic*, 50:510–530, 1985.

[CP99] Samir Chopra and Rohit Parikh. An inconsistency tolerant model for belief representation and belief revision. in *IJCAI 99*, to appear.

[DP94] Adnan Darwiche and Judea Pearl. On the logic of iterated belief revision. In *Proceedings of TARK 94*, 1994.

[Han92] Sven Ove Hansson. A dyadic representation of belief. In *Belief Revision*, Peter Gärdernfors, ed., Cambridge, 1992.

[GM88] P. Gärdenfors and D. Makinson. Revisions of knowledge systems using epistemic entrenchment. In *Proceedings of the Second Conference on Theoretical Aspects of Reasoning about Knowledge*, pages 661–672, 1988.

[Geo96] K. Georgatos. Ordering-based representations of rational inference. In J. J. Alferes, L. M. Pereira, and E. Orlowska, editors, *Logics in Artificial Intelligence (JELIA '96)*, number 1126 in Lecture Notes in Artificial Intelligence, pages 176–191, Berlin, 1996. Springer-Verlag.

[Geo97] K. Georgatos. Entrenchment relations: A uniform approach to nonmonotonic inference. In *Proceedings of the International Joint Conference on Qualitative and Quantitative Practical Reasoning (ESCQARU/FAPR 97)*, number 1244 in Lecture Notes in Computer Science, pages 282–297, Berlin, 1997. Springer-Verlag.

[Leh95] Daniel Lehmann. Belief revision, revised. In *Proceedings of Fourteenth International Joint Conference on Artificial Intelligence*, 1995, pages 1534-40.

[N97] A. C. Nayak, N.Y. Foo, M. Pagnucco, and A. Sattar. Changing conditional beliefs unconditionally In *Proceedings of TARK 96*, 1996. De Zeeuwse Stromen, The Netherlands, Morgan Kaufmann Publishers, 1966, pp 119-135.

[Neb92] Bernhard Nebel. Syntax based approaches to belief revision. In *Belief Revision*, Peter Gärdernfors, ed., Cambridge, 1992.

[PAR96] Rohit Parikh. Beliefs, belief revision, and splitting languages (presented at ITALLIC-96) In *Logic, Language, and Computation, Volume 2*, eds. Lawrence S. Moss, Jonathan Ginzburg, and Maarten de Rijke, CSLI Lecture Notes No. 96, CSLI Publications, 1999, pp. 266–268.