12.2 MINIMAL-SPANNING TREE TECHNIQUE

This minimal-spanning tree technique connects nodes at a minimum distance.

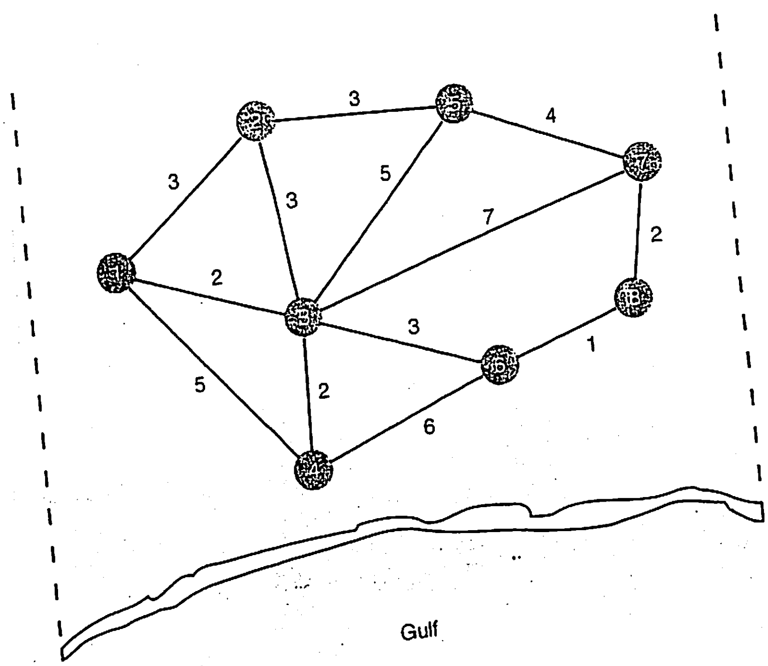
The minimal-spanning tree technique involves connecting all the points of a network together while minimizing the distance between them. It has been applied, for example, by telephone companies to connect a number of phones together while minimizing the total length of telephone cable.

Steps for the Minimal-Spanning Tree Technique

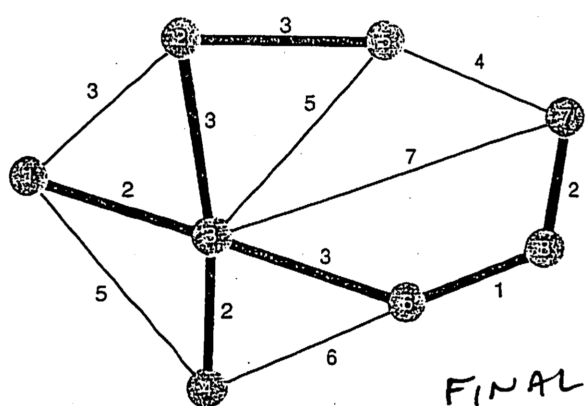
1. Select any node in the network.
2. Connect this node to the nearest node that minimizes the total distance.
3. Considering all of the nodes that are now connected, find and connect the nearest node that is not connected. If there is a tie for the nearest node, select one arbitrarily. A tie suggests there may be more than one optimal solution.
4. Repeat the third step until all nodes are connected.

Let us consider the Lauderdale Construction Company, which is currently developing a luxurious housing project in Panama City Beach, Florida. Melvin Lauderdale, owner and president of Lauderdale Construction, must determine the least expensive way to provide water and power to each house. The network of houses is shown in Figure 12.1.

As seen in Figure 12.1, there are eight houses on the gulf. The distance between each house in hundreds of feet is shown on the network. The distance between houses 1 and 2, for example, is 300 feet. (The number 3 is between nodes 1 and 2.) Now, the minimal-spanning tree technique is used to determine the minimal distance that can be used to connect all of the nodes. The approach is outlined as follows:



tions



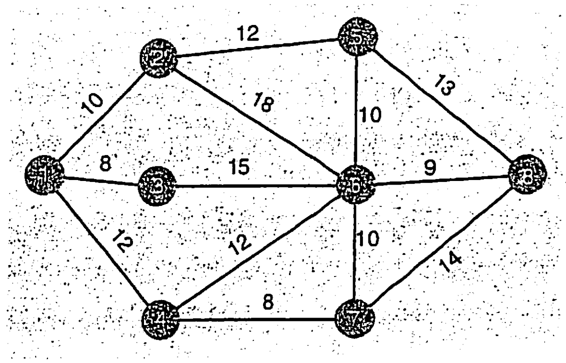
(*b*) Seventh Iteration

FINAL ANSWER 16

Solved Problem 12-1

Roxie LaMothe, owner of a large horse breeding farm near Orlando, is planning to install a complete water system connecting all of the various stables and barns. The location of the facilities and the distances between them is given in the network shown in Figure 12.15. Roxie must determine the least expensive way to provide water to each facility. What do you recommend?

FIGURE 12.15



Solution

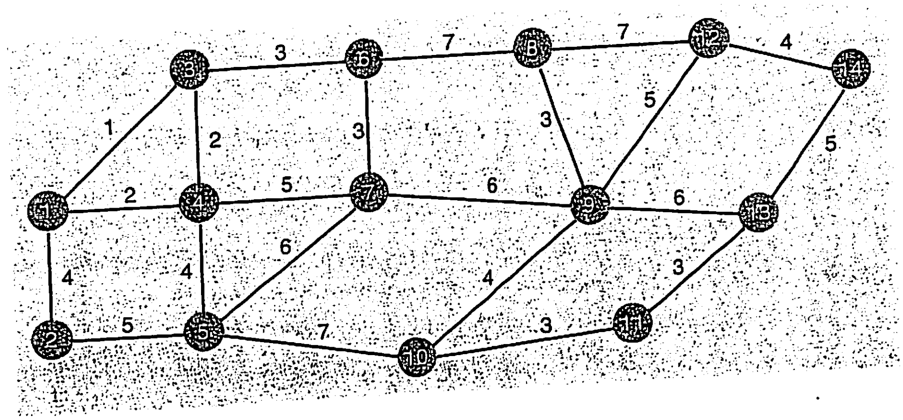
This is a typical minimum-spanning tree problem that can be solved by hand. We begin by selecting node 1 and connecting it to the nearest node, which is node 3. Nodes 1 and 2 are the next to be connected, followed by nodes 1 and 4. Now we connect node 4 to node 7 and node 7 to node 6. At this point, the only remaining points to be connected are node 6 to node 8 and node 6 to node 5. The final solution can be seen in Figure 12.16.

12-7 Bechtold Construction is in the process of installing power lines to a large housing development. Steve Bechtold wants to minimize the total length of wire used, which will minimize his costs. The housing development is shown as a network in Figure 12.21.

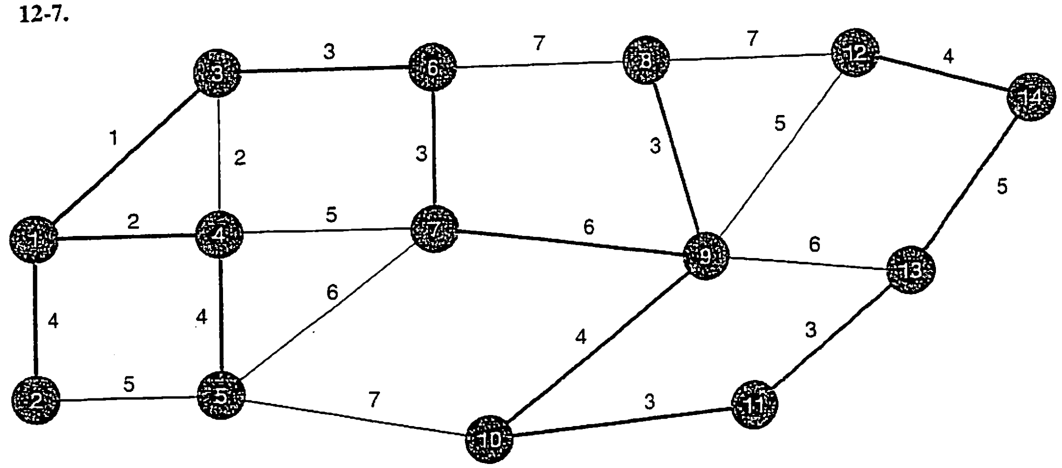
Each house has been numbered, and the distances between houses are given in hundreds of feet. What do you recommend?

FIGURE 12.21

Network for Problem 12-7



12-7.



THE BOOKS ANSWER

One optimal solution is shown. Connect 1–3, 1–4, 3–6, 6–7, 1–2, 4–5, 7–9, 8–9, 9–10, 10–11, 11–13, 13–14, and 12–14. Alternate solutions can be found by substituting 3–4 for 1–4 and substituting 9–12 for 13–14. Total distance = 45.

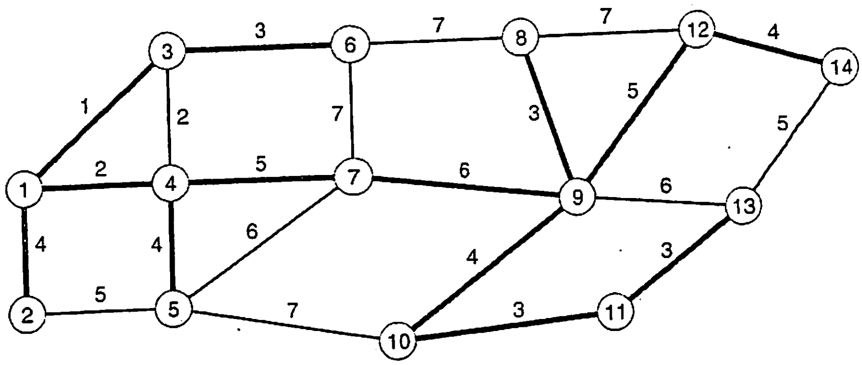
Page 5 of 18

• 12-10 Because of a sluggish economy, Bechtold Construction has been forced to modify its plans for the housing development in Problem 12-7. The result is that the path from node 6 to 7 now has a distance of 7. What impact does this have on the total length of wire needed to install the power lines?

THE BOOKS ANSWER

12-10. The minimal-spanning tree technique is needed to solve this problem. The minimum distance is 47 (4,700 feet). As you can see, the final solution has changed.

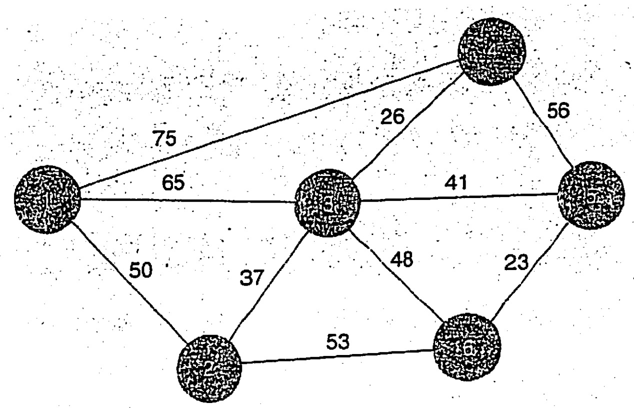
Figure for Problem 12-10



Q: 12-12 The director of security wants to connect security video cameras to the main control site from five potential trouble locations. Ordinarily, cable would simply be run from each location to the main control site. However, because the environment is potentially explosive, the cable must be run in a special conduit that is continually air purged. This conduit is very expensive but large enough to handle five cables (the maximum that might be needed). Use the minimal-spanning tree technique to find a minimum distance route for the conduit between the locations noted in Figure 12.24. (Note that it makes no difference which one is the main control site.)

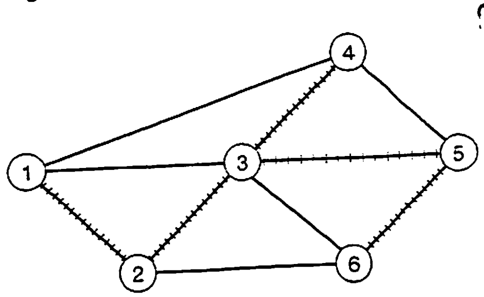
FIGURE 12.24

Network for Problem 12-12



12-12. This is the only optimum solution to this problem (177 units of length).

Books Answer



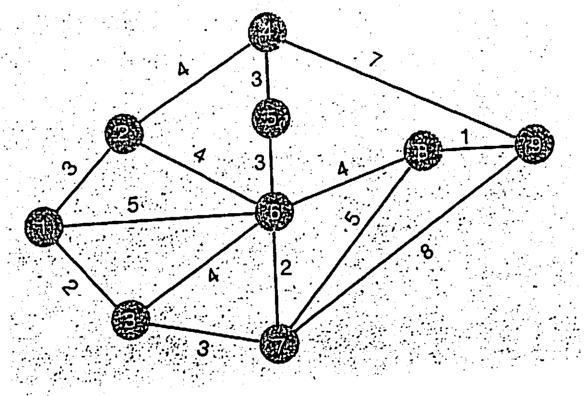
Page 7 of 18 Page 533

12-15 Solve the minimal-spanning tree problem in the network shown in Figure 12.26. Assume that the numbers in the network represent distance in hundreds of yards.

12-16 Refer to Problem 12-15. What impact would changing the value for path 6–7 to 500 yards have on the solution to the problem and the total distance?

FIGURE 12.26

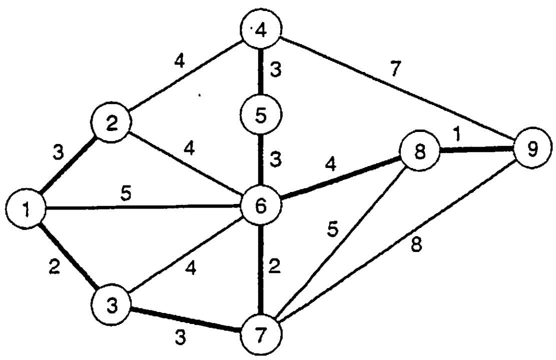
Network for Problem 12-15



BOOKS ANSWERS

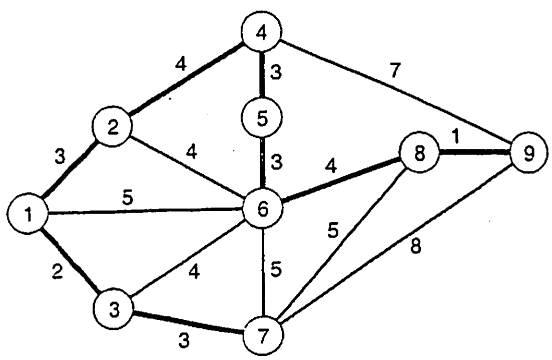
12-15. The solution to the minimal-spanning tree problem results in a minimum distance of 21 (2,100 yards). The final network follows.

Figure for Problem 12-15



12-16. If the distance between nodes 6 and 7 becomes 5, the minimum distance changes to 23 (2,300 yards). The final network follows. Another optimal solution exists.

Figure for Problem 12-16



12-26 The following table represents a network with the arcs identified by their starting and ending nodes. Draw the network and use the minimal spanning tree to find the minimum distance required to connect these nodes.

[illegible]	[illegible]
1–2	12
1–3	8
2–3	7
2–4	10
3–4	9
3–5	8
4–5	8
4–6	11
5–6	9

BOOKS ANSWER

12-26. Given the problem data, the network module in QM for Windows gives the following minimal spanning tree results. The branches 1–3, 2–3, 3–5, 4–5 and 5–6 are used to connect the nodes, and the total distance is 40.

Start node	End node	Cost	Include	Cost
1	2	12		
1	3	8	Y	8
2	3	7	Y	7
2	4	10		
3	4	9		
3	5	8	Y	8
4	5	8	Y	8
4	6	11		
5	6	9	Y	9
				40

12.4 SHORTEST-ROUTE TECHNIQUE

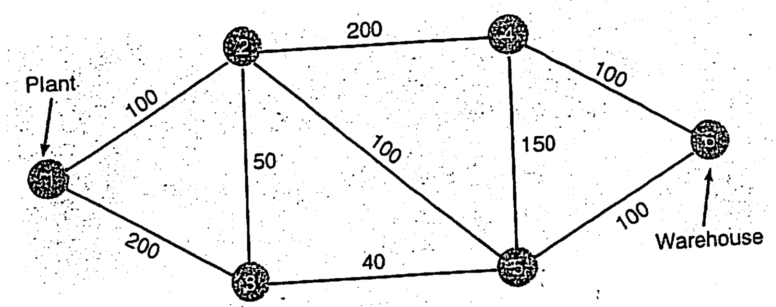
The shortest-route technique minimizes the distance through a network.

The *shortest-route technique* finds how a person or item can travel from one location to another while minimizing the total distance traveled. In other words, it finds the shortest route to a series of destinations.

Every day, Ray Design, Inc., must transport beds, chairs, and other furniture items from the factory to the warehouse. This involves going through several cities. Ray would like to find the route with the shortest distance. The road network is shown in Figure 12.10.

FIGURE 12.10

Roads from Ray's Plant to Warehouse



The steps of the shortest-route technique.

Steps of the Shortest-Route Technique

1. Find the nearest node to the origin (plant). Put the distance in a box by the node.
2. Find the next-nearest node to the origin (plant), and put the distance in a box by the node. In some cases, several paths will have to be checked to find the nearest node.
3. Repeat this process until you have gone through the entire network. The last distance at the ending node will be the distance of the shortest route. You should note that the distance placed in the box by each node is the shortest route to this node. These distances are used as intermediate results in finding the next-nearest node.

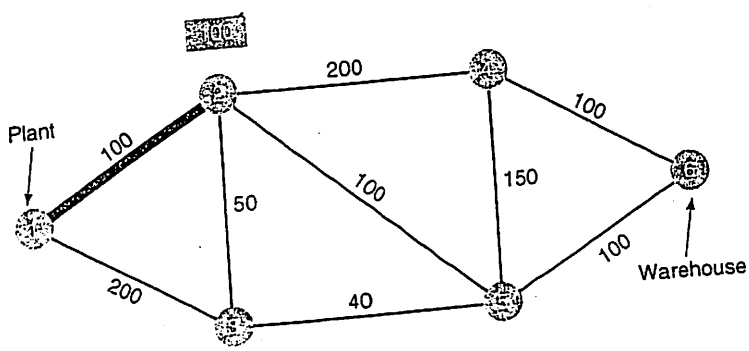
Looking at Figure 12.10, we can see that the nearest node to the plant is node 2, with a distance of 100 miles. Thus we will connect these two nodes. This first iteration is shown in Figure 12.11.

We look for the nearest node to the origin.

Now we look for the next-nearest node to the origin. We check nodes 3, 4, and 5. Node 3 is the nearest, but there are two possible paths. Path 1–2–3 is nearest to the origin, with a total distance of 150 miles (see Figure 12.12).

FIGURE 12.11

First Iteration for Ray Design



Pg 525 Page 10 of 18

FIGURE 12.12

Second Iteration for Ray Design

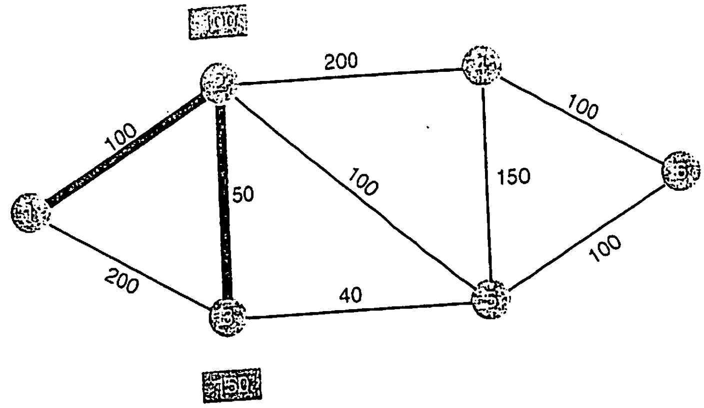


FIGURE 12.13

Third Iteration for Ray Design

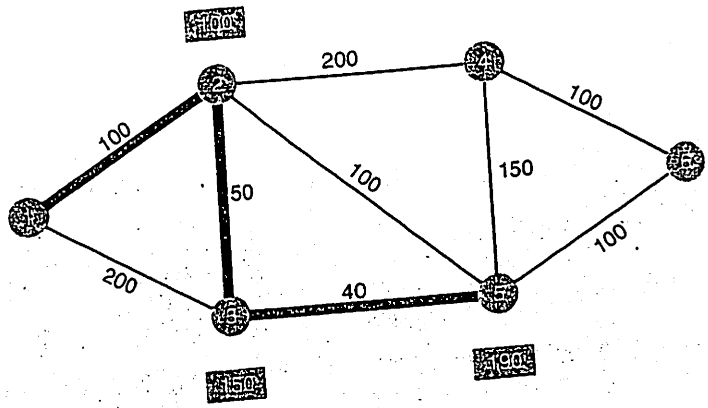
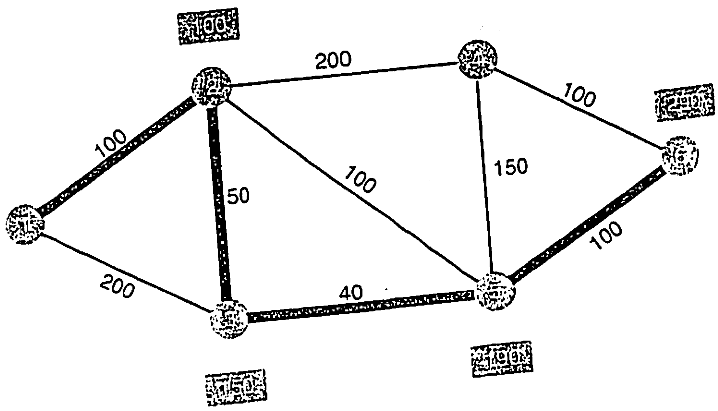


FIGURE 12.14

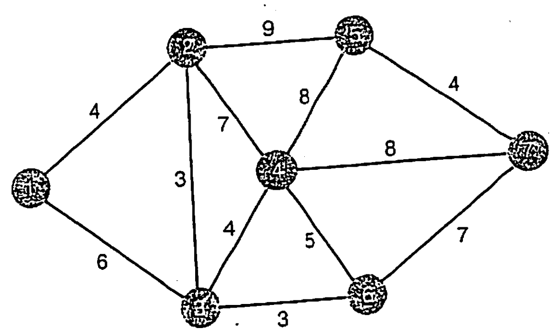
Fourth and Final Iteration for Ray Design



Q: 12-29 Use the shortest route algorithm to determine the minimum distance from node 1 to node 7 in Figure 12.31. Which nodes are included in this route?

FIGURE 12.31

Network for Problem 12-29



12-29. Using the shortest route technique in QM for Windows, we find the minimum total distance to be 16 as shown in the table.

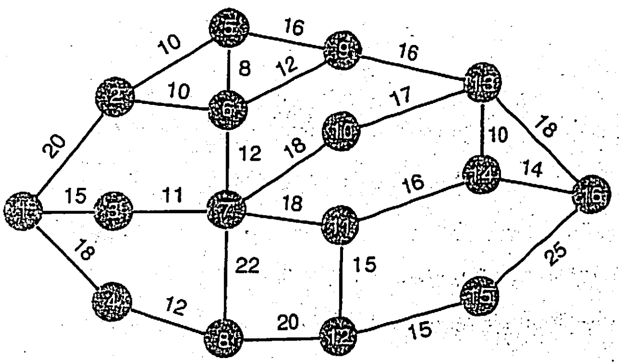
	Start Node	End Node	Distance	Cumulative Distance
Branch 2	1	3	6	6
Branch 7	3	6	3	9
Branch 12	6	7	7	16

12-21 Solve the shortest-route problem presented in the network of Figure 12.29 on the next page, going from node 1 to node 16. All numbers represent kilometers between German towns near the Black Forest.

FIGURE 12.29

Network for Problem 12-21

WOW
16 NODES



12-21. The shortest route from node 1 to node 16 is 74 kilometers. The solution along with the final network is shown in the following table and in the figure below.

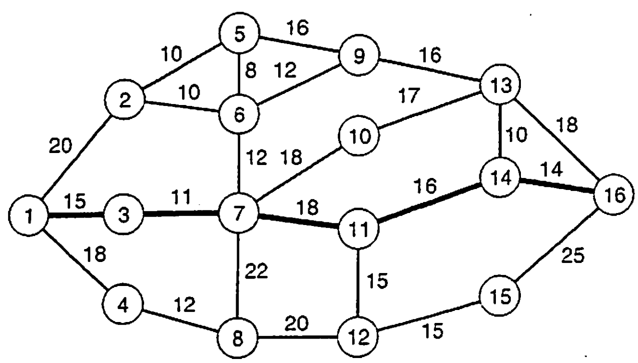
	Value
1–3	15
3–7	11
7–11	18
11–14	16
14–16	14

Shortest path:
1–3–7–11–14–16

Total shortest distance: 74.

NOTE:
ANSWER
DOES NOT
go thru all
nodes

Figure for Problem 12-21

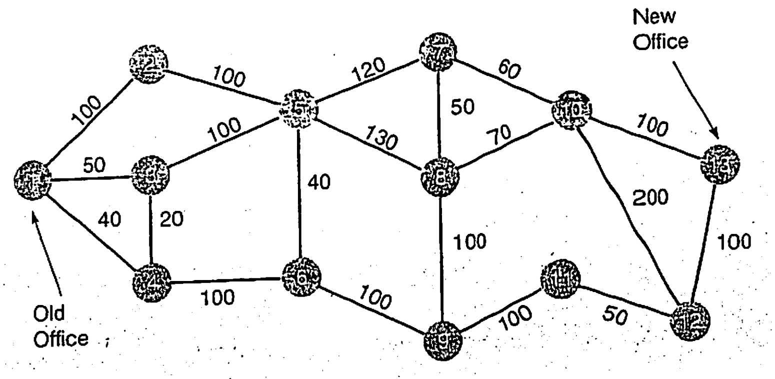


Page 13 of 18

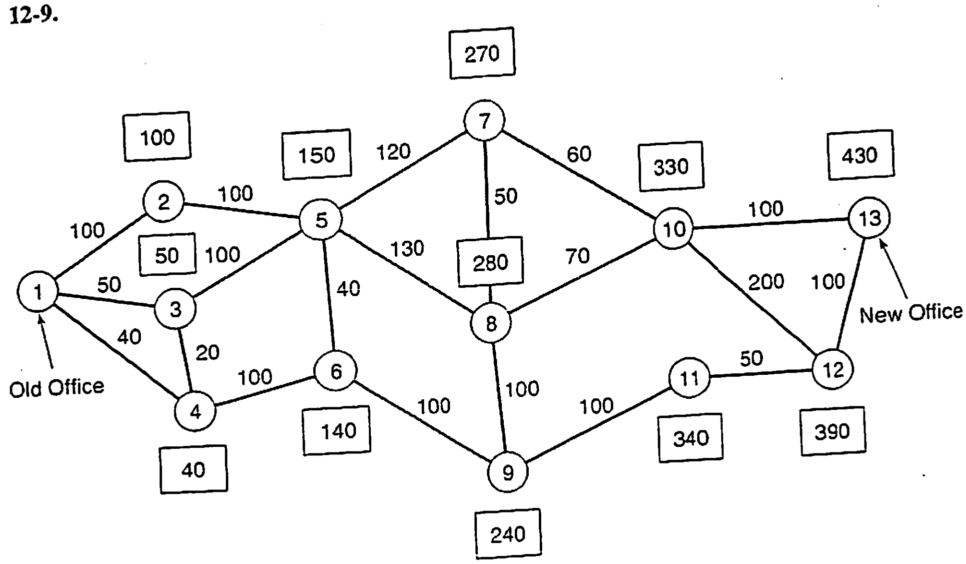
12-9 Transworld Moving has been hired to move the office furniture and equipment of Cohen Properties to their new headquarters. What route do you recommend? The network of roads is shown in Figure 12.23.

FIGURE 12.23

Network for Problem 12-9



12-9.

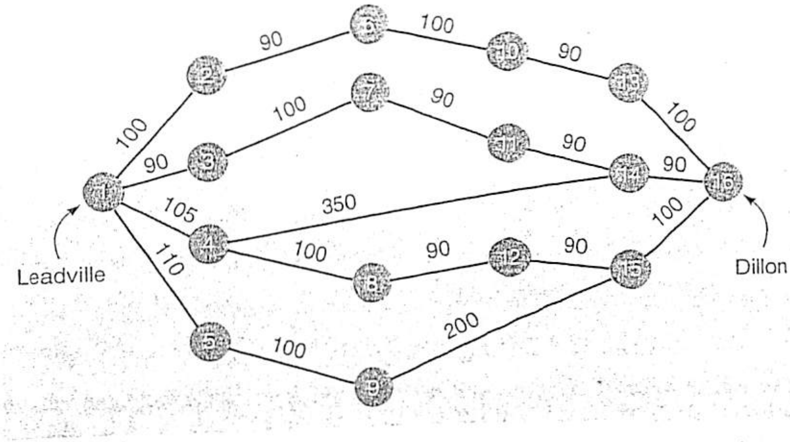


The shortest route is 1–3–5–7–10–13. The distance is 430 miles.

Solved Problem 12-3

The network of Figure 12.19 shows the highways and cities surrounding Leadville, Colorado. Leadville Tom, a bicycle helmet manufacturer, must transport his helmets to a distributor based in Dillon, Colorado. To do this, he must go through several cities. Tom would like to find the shortest way to get from Leadville to Dillon. What do you recommend?

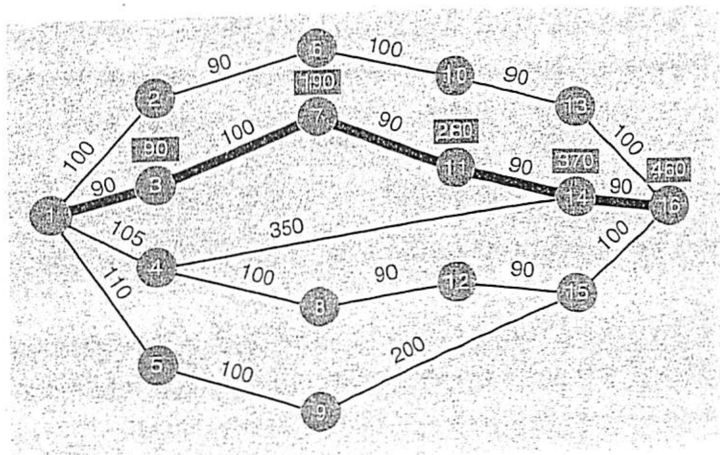
FIGURE 12.19



Solution

This problem can be solved using the shortest-route technique discussed in the chapter. The nearest node to the origin (Leadville) is node 3, with a distance of 90 miles. Thus, we put 90 in a box by node 3. The next-nearest node to the origin is node 7 at 190 miles. Again, we put 190 in a box by node 7. Next is node 11 at 280 miles and then node 14 at 370 miles. Finally, we see that the next-nearest (and final) node is node 16 at 460 miles. See Figure 12.20 for the solution.

FIGURE 12.20



Page 15 of 18

Q: 12-22 Due to bad weather, the roads represented by nodes 7 and 8 have been closed (see Problem 12-21). No traffic can get onto or off of these roads. Describe the impact that this will have (if any) on the shortest route through this network.

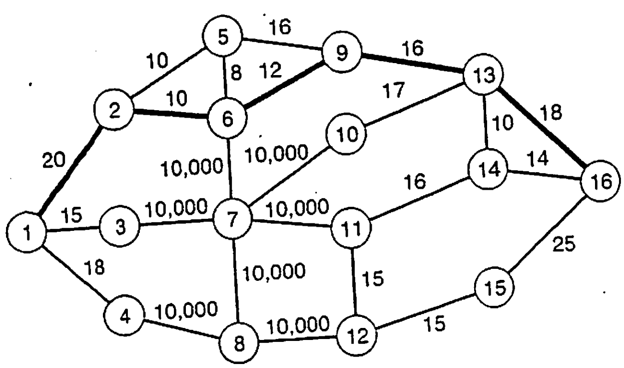
12-22. The impact of closing two nodes (nodes 7 and 8) is to increase the shortest route from 74 to 76 kilometers. Note that all paths into and from nodes 7 and 8 have their values changed to a very high relative number (10,000) to force these paths out of the final solution. The solution along with the final network is given below.

	Value
1–2	20
2–6	10
6–9	12
9–13	16
13–16	18

Shortest path:
1-2-6-9-13-16

Total shortest distance: 76.

Figure for Problem 12-22



12-24 In going from Quincy to Old Bainbridge, there are 10 possible roads that George Olin can take. Each road can be considered a branch in the shortest-route problem.

(a) Determine the best way to get from Quincy (node 1) to Old Bainbridge (node 8) that will minimize total distance traveled. All distances are in hundreds of miles.

[illegible]	[illegible]	[illegible]	[illegible]
Branch 1	1	2	3
Branch 2	1	3	2
Branch 3	2	4	3
Branch 4	3	5	3
Branch 5	4	5	1
Branch 6	4	6	4
Branch 7	5	7	2
Branch 8	6	7	2
Branch 9	6	8	3
Branch 10	7	8	6

(b) George Olin made a mistake in estimating the distances from Quincy to Old Bainbridge. The new distances are in the following table. What impact does this have on the shortest route from Quincy to Old Bainbridge?

Branch	Start Node	End Node	Distance (in Hundreds of Miles)
Branch 1	1	2	3
Branch 2	1	3	2
Branch 3	2	4	3
Branch 4	3	5	1
Branch 5	4	5	1
Branch 6	4	6	4
Branch 7	5	7	2
Branch 8	6	7	2
Branch 9	6	8	3
Branch 10	7	8	6

12-24. a. Using the shortest-route technique, George can determine the best way to go from Quincy to Old Bainbridge. The data and results are below. As can be seen, the shortest route is to take branches 2, 4, 7, 8 and 9 with a minimum distance of 1,200 miles.

	Start Node	End Node	Distance
Branch 1	1	2	3
Branch 2	1	3	2
Branch 3	2	4	3
Branch 4	3	5	3
Branch 5	4	5	1
Branch 6	4	6	4
Branch 7	5	7	2
Branch 8	6	7	2
Branch 9	6	8	3
Branch 10	7	8	6

Shortest Path
Total distance = 12

	Start Node	End Node	Distance	Cumulative Distance
Branch 2	1	3	2	2
Branch 4	3	5	3	5
Branch 7	5	7	2	7
Branch 8	7	6	2	9
Branch 9	6	8	3	12

Minimum distance matrix

	Node 1	Node 2	Node 3	Node 4
Node 1	0	3	2	6
Node 2	3	0	5	3
Node 3	2	5	0	4
Node 4	6	3	4	0
Node 5	5	4	3	1
Node 6	9	7	7	4
Node 7	7	6	5	3
Node 8	12	10	10	7

	Node 5	Node 6	Node 7	Node 8
Node 1	5	9	7	12
Node 2	4	7	6	10
Node 3	3	7	5	10
Node 4	1	4	3	7
Node 5	0	4	2	7
Node 6	4	0	2	3
Node 7	2	2	0	5
Node 8	7	3	5	0

12-24. b. George can use the shortest-route model to determine the impact of the changes. The results are below. As you can see, the new shortest route is 1,000 miles (called 10 in the printout since units are in 100's).

	Start Node	End Node	Distance
Branch 1	1	2	3
Branch 2	1	3	2
Branch 3	2	4	3
Branch 4	3	5	1
Branch 5	4	5	1
Branch 6	4	6	4
Branch 7	5	7	2
Branch 8	6	7	2
Branch 9	6	8	3
Branch 10	7	8	6

Shortest Path
Total distance = 10

	Start Node	End Node	Distance	Cumulative Distance
Branch 2	1	3	2	2
Branch 4	3	5	1	3
Branch 7	5	7	2	5
Branch 8	7	6	2	7
Branch 9	6	8	3	10

Minimum distance matrix

	Node 1	Node 2	Node 3	Node 4
Node 1	0	3	2	4
Node 2	3	0	5	3
Node 3	2	5	0	2
Node 4	4	3	2	0
Node 5	3	4	1	1
Node 6	7	7	5	4
Node 7	5	6	3	3
Node 8	10	10	8	7

	Node 5	Node 6	Node 7	Node 8
Node 1	3	7	5	10
Node 2	4	7	6	10
Node 3	1	5	3	8
Node 4	1	4	3	7
Node 5	0	4	2	7
Node 6	4	0	2	3
Node 7	2	2	0	5
Node 8	7	3	5	0

Page 18 of 18

SELF-TEST

- Before taking the self-test, refer to the learning objectives at the beginning of the chapter, the notes in the margins, and the glossary at the end of the chapter.
- Use the key at the back of the book to correct your answers.
- Restudy pages that correspond to any questions that you answered incorrectly or material you feel uncertain about.

1. Which technique is used to connect all points of a network together while minimizing the distance between them?
   a. maximal flow
   b. minimal flow
   c. minimal-spanning tree
   d. shortest route
   e. longest span
2. The first step of the minimal-spanning tree technique is to
   a. select the node with the highest distance between it and any other node.
   b. select the node with the lowest distance between it and any other node.
   c. select the node that is closest to the origin.
   d. select any arc that connects two nodes.
   e. select any node.
3. The first step of the maximal-flow technique is to
   a. select any node.
   b. pick any path from the start to the finish with some flow.
   c. pick the path with the maximum flow.
   d. pick the path with the minimal flow.
   e. pick a path where the flow going into each node is greater than the flow coming out of each node.
4. In which technique do you connect the nearest node to the existing solution that is not currently connected?
   a. maximal tree
   b. shortest route
   c. minimal-spanning tree
   d. maximal flow
   e. minimal flow
5. In the shortest-route technique, the objective is to determine the route from an origin to a destination that passes through the fewest number of other nodes.
   a. True
   b. False
6. Adjusting the flow capacity numbers on a path is an important step in which technique?
   a. maximal flow
   b. minimal flow
   c. maximal-spanning tree
   d. minimal-spanning tree
   e. shortest route
7. When the optimal solution has been reached with the maximal-flow technique, every node will be connected with at least one other node.
   a. True
   b. False
8. A large city is planning for delays during rush hour when roads are closed for maintenance. On a normal weekday, 160,000 vehicles travel on a freeway from downtown to a point 15 miles to the west. Which of the techniques discussed in this chapter would help the city planners determine if alternate routes provide sufficient capacity for all the traffic?
   a. minimal spanning tree technique
   b. maximal-flow technique
   c. shortest-route technique
9. The computing center at a major university is installing new fiber optic cables for a campuswide computer network. Which of the techniques in this chapter could be used to determine the least amount of cable needed to connect the 20 buildings on campus?
   a. minimal spanning tree technique
   b. maximal-flow technique
   c. shortest-route technique
10. In a minimal spanning tree problem, the optimal solution has been found when
    a. the start node and the finish node are connected by a continuous path.
    b. the flow from the start node is equal to the flow into the finish node.
    c. all arcs have been selected to be a part of the tree.
    d. all nodes have been connected and are a part of the tree.
11. ______________ is a technique that is used to find how a person or an item can travel from one location to another while minimizing the total distance traveled.
12. The technique that allows us to determine the maximum amount of a material that can flow through a network is called ______________.
13. The ______________ technique can be used to connect all of the points of a network together while minimizing the distance between them.

1 C
2 E
3
4 C
5 FALSE
6 A
7
8 B
9 A
10 D
11 short route
12 max flow
13 min span