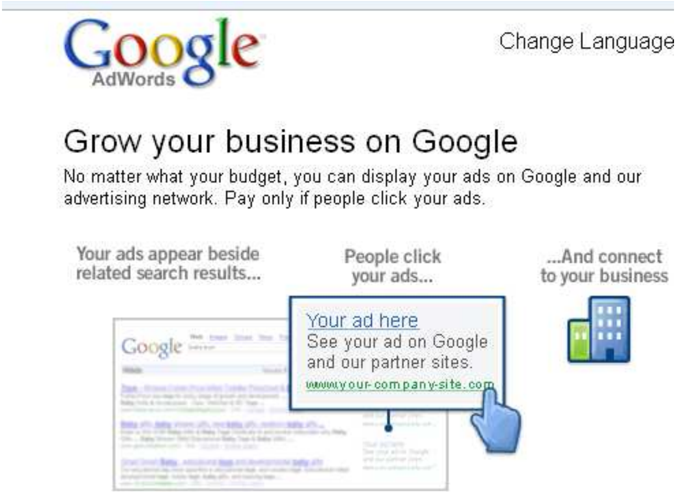
Adwords, An Algorithmic Perspective

Shoshana Neuburger

April 20, 2009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AdWords
Change Language
Grow your business on Google
No matter what your budget, you can display your ads on Google and our advertising network. Pay only if people click your ads.
Your ads appear beside related search results...
People click your ads...
...And connect to your business
Your ad here
See your ad on Google and our partner sites.
www.your-company-site.com

What are Adwords?

- Search engine displays search results.

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- Search engine displays search results.
- For each query, relevant ads are also returned.

What are Adwords?

- Search engine displays search results.
- For each query, relevant ads are also returned.
- The search results and ads are displayed separately.

Some searches reveal many sponsored ads...

4/8/2009 college furniture - Google Search

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Google [college furniture] Search Advanced Search Preferences

Web Results **1** - **10** of about **43,000,000** for **college furniture**. (**0.31** seconds)

College and Dorm room **Furniture** and Rug Boston Massachusetts
Over 30 years **College Furniture** has supplied Boston Massachusetts colleges with dorm room **furniture** and the area with inexpensive **furniture**.
www.**collegefurniture**cheap.com/ - 11k - Cached - Similar pages -

College Furniture & Dorm Room **Furniture** - Free Shipping
College Furniture & Dorm Room **Furniture** The place to find great deals for dorm room **furniture**. We offer a wide selection and variety of **college** dorm room ...
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Shop for Dorm Essentials **College** Home Products and Promotions at Target. ... Featured Items in **College Furniture** and Dorm Essentials. **College Furniture** and ...
www.target.com/**Furniture**-Dorm-Essentials-**College**-Home/b?ie=UTF8&node=360124011 - 188k - Cached - Similar pages -

Dorm **Furniture** - **College** Bunk Beds and Computer Desks
Dorm Room Station offers sale priced **college furniture**, computer desks and bunk beds. Secure online shopping. Direct delivery.
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Dorm **Furniture**
College Dorm Living **Furniture** Seating, Chairs, Futons, & Bedding!
www.Dormbuys.com

Buy Dorm Room **Furniture**
Studio Sofas Starting at just $325 Great Selection plus Free Shipping!
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Dorm **Furniture**
Finest Solid Wood **Furniture** Built to Last, Quick Delivery!
www.JessCrate.com

College Furniture
Head to **College** in Style with Dorm

And others result in none...

4/20/2009 shoshana neuburger - Google Search

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Web Results **1** - **10** of about **1,140** for **shoshana neuburger**. (**0.22** seconds)

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Advertiser

- An advertiser is charged for his ads.

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- An advertiser is charged for his ads.
- Each advertiser can value each keyword differently.
- An advertiser bids for a keyword that should display his ad.
- Some keywords are more popular among advertisers.

Search engine's perspective:

- Maximize revenue each day.
- Limitations:
  - must respect each advertiser's daily budget
    and
  - the profit depends on an advertiser's bid for a keyword he wins.

The bids and daily budgets are specified in advance.

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- Maximize revenue each day.
- Limitations:
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Decision: which ads to display for a query?

Online Algorithm

The Adwords problem is an online allocation problem.

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ALG is α-competitive if the ratio between its performance and the optimal offline performance is bounded by α.

$\frac{ALG(I)}{OPT(I)} \geq \alpha$ for all instances I.

Greedy Criteria

Maximize the profit for each query.

As a keyword arrives, choose the ad that offers the highest bid.

Until the advertiser's budget is depleted.

Greedy Algorithm

Example

	$Bidder_1$	$Bidder_2$
Bicycle	\$1	\$0.99
Flowers	\$1	\$0
Budget	\$100	\$100

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Queries:

100 Bicycles
then
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Greedy allocates 100 Bicycles to $Bidder_1$.

Greedy Algorithm

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	$Bidder_1$	$Bidder_2$
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Queries:

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OPT (offline) allocates 100 Bicycles to $Bidder_2$ and 100 Flowers to $Bidder_1$.

Greedy Algorithm

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Bicycle	\$1	\$0.99
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Budget	\$100	\$100

Queries:

100 Bicycles
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Algorithm	Allocation	Revenue
Greedy	$Bidder_1$: 100 Bicycles	\$100
OPT	$Bidder_1$: 100 Flowers	
	$Bidder_2$: 100 Bicycles	\$199

Competitive Analysis

ALG is α-competitive if the ratio between its performance and the optimal offline performance is bounded by α.

$\frac{ALG(I)}{OPT(I)} \geq \alpha$ for all instances I.

Algorithm	α	
Greedy	$\frac{1}{2}$	
Mehta, Saberi, Vazirani, Vazirani ('05)	$1 - \frac{1}{e} \approx .63$	optimal

AdWords and Generalized On-line Matching

Aranyak Mehta, Amin Saberi, Umesh Vazirani, Vijay Vazirani
FOCS, 2005

In search of a better algorithm

As a query arrives, the algorithm

- Should favor advertisers with high bids
- Should not exhaust the budget of any advertiser too quickly

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The algorithm weighs the remaining fraction of each advertiser's budget against the amount of its bid.

Previous Results

Karp, Vazirani, Vazirani (1990)

- Online bipartite matching
- Randomized algorithm RANKING
- Fixes random permutation of bidders in advance.
- Budgets $= 1$, Bids $= 0/1$
- Factor: $1 - \frac{1}{e}$

Previous Results

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  RANKING
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Kalyanasundaram, Pruhs (2000)

- Online b-matching
- Deterministic algorithm
  BALANCE
- Matches query to bidder with highest remaining budget.
- Budgets $= 1$, Bids $= 0/\epsilon$
- Factor: $1 - \frac{1}{e}$

KVV '90

"We saw online matching as a beautiful research problem with purely theoretical appeal. At the time, we had no idea it would turn out to have practical value."

- Umesh Vazirani, SIAM News, April 2005

Applying BALANCE

The new algorithm generalizes b-matching to arbitrary bids.

Natural Algorithm:

- Assign query to highest bidder
- Break ties with largest remaining budget

Achieves competitive ratio $< 1 - \frac{1}{e}$.

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We would like to do better!

Adwords problem:

- N bidders
- Each bidder i has daily budget b_i
- Each bidder i specifies a bid c_{iq} for query word $q \in Q$
- A sequence of query words $q_1 q_2 \cdots q_M$, $q_j \in Q$, arrive online during the day.
- Each query q_j must be assigned to some bidder i as it arrives; the revenue is c_{iq_j}

Objective: maximize total daily revenue while respecting daily budget of bidders.

Simplified version of Adwords problem:

- Bidder pays as soon as ad is displayed
- Bidder pays his own bid
- One ad displayed per search page

Assumption: bids are small compared to budgets.

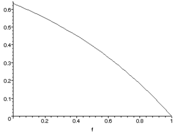
New Algorithm

Algorithm: Give query to bidder that maximizes

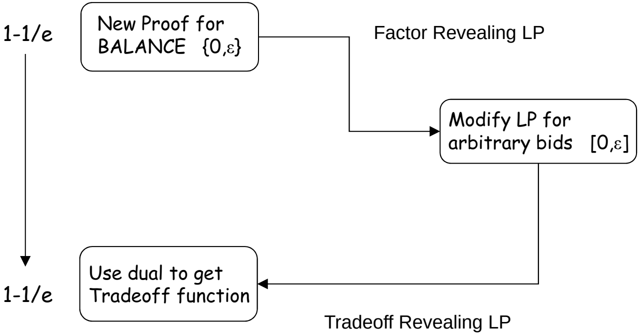
$$\textit{bid} \times \psi(\textit{fraction of budget spent})$$

ψ is tradeoff function between bid and unspent budget.

$$\psi(x) = 1 - e^{-(1-x)}$$



Where does ψ come from?



Factor-Revealing LP

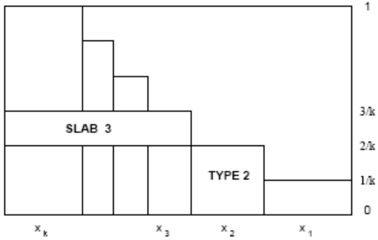
Choose large k.

Discretize the budget of each bidder into k equal *slabs*.

Define the *type* of a bidder by the fraction of budget spent at end of BALANCE.

Define $x_1, x_2, \ldots, x_k$:
$x_i =$ number of bidders of type i.

Factor-Revealing LP



w.l.o.g. OPT = \$N.

$$\text{Revenue} = \sum_{i=1}^{k} x_i \frac{i}{k}$$

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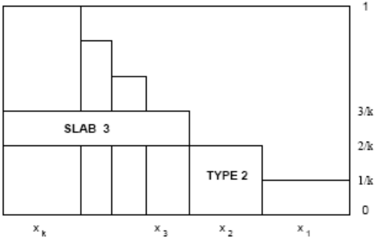

w.l.o.g. OPT = \$N.

$$\text{Revenue} = \sum_{i=1}^{k} x_i \frac{i}{k}$$

Constraint 1: $x_1 \leq \frac{N}{k}$

Constraint 2: $x_1 + x_2 \leq 2\frac{N}{k} - \frac{x_1}{k}$

$$\forall i, 1 \leq i \leq k-1: \sum_{j=1}^{i} (1 + \frac{i-j}{k}) x_j \leq \frac{i}{k} N$$

Factor-Revealing LP

$$\begin{array}{ll} \text{Minimize} & \displaystyle\sum_{i=1}^{k} x_i \frac{i}{k} \\ \text{such that} & \displaystyle\sum_{j=1}^{i} \left(1 + \frac{i-j}{k}\right) x_j \leq \frac{i}{k} N \\ & \displaystyle\sum_{j} x_j = N \end{array}$$

Factor-Revealing LP

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Solve LP by finding the opimum primal and dual.
Optimal solution is $x_i = \frac{N}{k}(1 - \frac{1}{k})^{i-1}$
which tends to $N(1 - \frac{1}{e})$ as $k \to \infty$.
Thus, BALANCE achieves a factor of $1 - \frac{1}{e}$.

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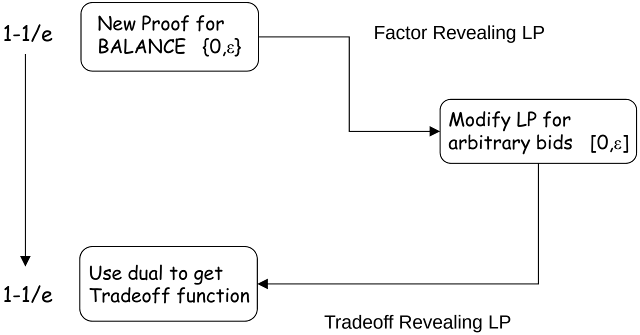

Modify the LP for arbitrary bids

Subtle tradeoff between bid and unspent budget

We generalize LP L and its dual D to the case with arbitrary bids using LPs $L(\pi, \psi)$.

L: $\begin{array}{c} \text{Max } c \cdot x \\ Ax \leq b \end{array}$	D: $\begin{array}{c} \text{Min } b \cdot y \\ y^T A \geq c \end{array}$
$L(\pi, \psi)$ $\begin{array}{c} \text{Max } c \cdot x \\ Ax \leq b + \Delta(\pi, \psi) \end{array}$	$D(\pi, \psi)$ $\begin{array}{c} \text{Min } b \cdot y + \Delta(\pi, \psi) \cdot y \\ y^T A \geq c \end{array}$

Modify the LP for arbitrary bids

$L(\pi, \psi)$	$D(\pi, \psi)$
Max $c \cdot x$	Min $b \cdot y + \Delta(\pi, \psi) \cdot y$
$Ax \leq b + \Delta(\pi, \psi)$	$y^T A \geq c$

Observation: For every ψ, dual achieves optimal value at same vertex.

Modify the LP for arbitrary bids

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There is a way to choose ψ so that the objective function does not decrease.
Thus, the competitive factor remains $1 - \frac{1}{e}$.

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Thus, the competitive factor remains $1 - \frac{1}{e}$.

This competitive factor is optimal.

More Realistic Models

The algorithm introduced by MSVV generalizes to

- Advertisers with different daily budgets.
- The optimal allocation does not exhaust all budgets.
- Several ads per search query.
- Cost-per-Click
- Second Price

Online budgeted matching in random input models with applications to Adwords

Gagan Goel and Aranyak Mehta
SODA, 2008

Outline of article

- Distributional assumption about query sequence: although the set of queries is arbitrary, the order of queries is random.
- Main Result: Greedy has competitive ratio $1 - \frac{1}{e}$ in the random permutation input model
- This result applies to the i.i.d. model as well.
- Approach: modify KVV (fix hole in proof) and then apply results to Adwords problem

Permutation Classes

For each item p, classify permutations

- Into those in which p remains unmatched, *miss*.
- Into those in which p gets matched.
  Subclasses depending on the structure of the match:
  - *good*: the correct match is available when p arrives
  - *bad*: the correct match is allocated prior to the arrival of p

GM Algorithm

- Properties of Greedy:
  - Monotonicity
  - Prefix
  - Partition
- These properties are simple observations in bipartite matching.
- There can be several different bids for the same query in the Adwords problem.
- Not every mismatch can be reversed easily; a tagging procedure is used to generalize the results.
- The tagging method works on permutations of the input.

GM Algorithm

$$\text{Revenue} = \sum_{i=1}^{m} min \left\{ B_i, \sum_{q \in Q} bid_{iq} \right\}$$

In the last bid the algorithm allocates to a bidder, his budget may be exceeded.

Inequalities bound the sizes of miss, bad, good.

A linear program maximizes the loss of revenue over these inequalities.

Factor revealing LP proves competitive ratio of $1 - \frac{1}{e}$ in random input model.

GM Results

Competitive factor of Greedy in

- random-permutation input model
- independent distribution input model (i.i.d.)

is exactly $1 - \frac{1}{e}$.

Thank you!