# LEGGED ROBOTS

*Research on legged machines can lead to the construction of useful legged vehicles and help us to understand legged locomotion in animals.*

**MARC H. RAIBERT**

## WHY STUDY LEGGED MACHINES?

Aside from the sheer thrill of creating machines that actually run, there are two serious reasons for exploring the use of legs for locomotion. One is mobility: There is a need for vehicles that can travel in difficult terrain, where existing vehicles cannot go. Wheels excel on prepared surfaces such as rails and roads, but perform poorly where the terrain is soft or uneven. Because of these limitations, only about half the earth's landmass is accessible to existing wheeled and tracked vehicles, whereas a much greater area can be reached by animals on foot. It should be possible to build legged vehicles that can go to the places that animals can now reach.

One reason legs provide better mobility in rough terrain is that they can use isolated footholds that optimize support and traction, whereas a wheel requires a continuous path of support. As a consequence, a legged system can choose among the best footholds in the reachable terrain; a wheel must negotiate the worst terrain. A ladder illustrates this point: Rungs provide footholds that enable the ascent of legged systems, but the spaces between the rungs prohibit the ascent of wheeled systems.

Another advantage of legs is that they provide an active suspension that decouples the path of the body from the paths of the feet. The payload is free to travel smoothly despite pronounced variations in the terrain. A legged system can also step over obstacles. In principle, the performance of legged vehicles can, to a great extent, be independent of the detailed roughness of the ground.

The construction of useful legged vehicles depends on progress in several areas of engineering and science. Legged vehicles will need systems that control joint motions, sequence the use of legs, monitor and manipulate balance, generate motions to use known footholds, sense the terrain to find good footholds, and calculate negotiable foothold sequences. Most of these tasks are not well understood yet, but research is under way. If this research is successful, it will lead to the development of legged vehicles that travel efficiently and quickly in terrain where softness, grade, or obstacles make existing vehicles ineffective. Such vehicles will be useful in industrial, agricultural, and military applications.

The second reason for exploring legged machines is to gain a better understanding of human and animal locomotion. Slow-motion television replays reveal to us the large variety and complexity of ways athletes can carry, swing, toss, glide, and otherwise

---

With the exception of a few modifications, this article is excerpted from *Legged Robots that Balance*.

propel their bodies through space, maintaining orientation, balance, and speed as they go. Such performance is not limited to professional athletes; behavior at the local playground is equally impressive from a mechanical engineering, sensory–motor integration, or computational point of view. Animals also demonstrate great mobility and agility. They use their legs to move quickly and reliably through forest, swamp, marsh, and jungle, and from tree to tree. Sometimes they move with great speed, often with great efficiency.

Despite the skill we apply in using our own legs for locomotion, we are still at a primitive stage in understanding the control principles that underlie walking and running. What control mechanisms do animals use? One way to learn more about plausible mechanisms for animal locomotion is to build legged machines. To the extent that an animal and a machine perform similar locomotion tasks, their control systems and mechanical structures must solve similar problems. By building machines, we can gain new insights into these problems, and learn about possible solutions. Of particular value is the rigor required to build physical machines that actually work. The concrete theories and algorithms developed for such machines can guide biological research by suggesting specific models for experimental testing and verification. This sort of interdisciplinary approach is already becoming popular in other areas where biology and robotics have a common ground, such as vision, speech, and manipulation.

## RESEARCH ON LEGGED MACHINES

The scientific study of legged locomotion began just over a century ago when Leland Stanford, then governor of California, commissioned Eadweard Muybridge to find out whether or not a trotting horse left the ground with all four feet at the same time. See Table I for milestones in the development of legged robots. Stanford had wagered that it never did. After Muybridge proved him wrong with a set of stop-motion photographs that appeared in *Scientific American* in 1878, Muybridge went on to document the walking and running behavior of over 40 mammals, including humans [24, 25]. His photographic data are still of considerable value and survive as a landmark in locomotion research.

The study of machines that walk also had its origin in Muybridge's time. An early walking model appeared in about 1870 [13]. It used a linkage to move the body along a straight horizontal path while the feet moved up and down to exchange support during stepping (see Figure 1). The linkage was originally designed by the famous Russian mathematician Chebyshev some years earlier. During the 80 or 90 years that followed, workers viewed the task of building walking machines as the task of designing linkages that would generate suitable stepping motions when driven by a source of power. Many designs were proposed (e.g., [1, 21, 34, 36, 38]), but the performance of such machines was limited by their fixed patterns of motion, since they could not adjust to variations in the terrain by placing the feet on the

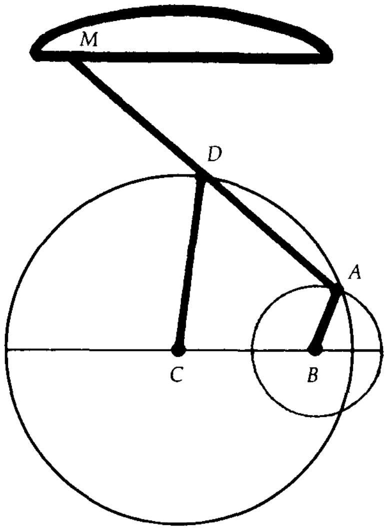
**TABLE I. Milestones in the Development of Legged Robots**

| | | |
|---|---|---|
| 1850 | Chebyshev | Designs linkage used in early walking mechanism [13]. |
| 1872 | Muybridge | Uses stop-motion photography to document running animals. |
| 1893 | Rygg | Patents human-powered mechanical horse. |
| 1945 | Wallace | Patents hopping tank with reaction wheels that provide stability. |
| 1961 | Space General | Eight-legged kinematic machine walks in outdoor terrain [21]. |
| 1963 | Cannon, Higdon, and Schaefer | Control system balances single, double, and limber inverted pendulums. |
| 1968 | Frank and McGhee | Simple digital logic controls walking of Phony Pony. |
| 1968 | Mosher | GE quadruped truck climbs railroad ties under control of human driver. |
| 1969 | Bucyrus-Erie Co. | Big Muskie, a 15,000-ton walking dragline, is used for strip mining. It moves in soft terrain at a speed of 900 ft./h. [35]. |
| 1977 | McGhee | Digital computer coordinates leg motions of hexapod walking machine. |
| 1977 | Gurfinkel | Hybrid computer controls hexapod walker in USSR. |
| 1977 | McMahon and Greene | Human runners set new speed records on tuned track at Harvard. Its compliance is adjusted to mechanics of human leg. |
| 1980 | Hirose and Umetani | Quadruped machine climbs stairs and climbs over obstacles using simple sensors. The leg mechanism simplifies control. |
| 1980 | Kato | Hydraulic biped walks with quasi-dynamic gait. |
| 1980 | Matsuoka | Mechanism balances in the plane while hopping on one leg. |
| 1981 | Miura and Shimoyama | Walking biped balances actively in three-dimensional space. |
| 1983 | Sutherland | Hexapod carries human rider. Computer, hydraulics, and human share computing task. |
| 1983 | Odetics | Self-contained hexapod lifts and moves back end of pickup truck [31]. |

best footholds (see Figure 2, page 502). By the late 1950s, it had become clear that linkages providing fixed motion would not suffice and that useful walking machines would need *control* [11].

One approach to control was to harness a human. Ralph Mosher used this approach in building a four-legged walking truck at General Electric in the mid 1960s [12]. The project was part of a decade-long campaign to build advanced teleoperators, capable of providing better dexterity through high-fidelity force feedback. The machine Mosher built stood 11 feet tall, weighed 3000 pounds, and was powered hydraulically. It is shown in Figure 3, page 503. Each of the driver's limbs was connected to a handle or pedal that controlled one of the truck's four legs. Whenever the driver caused a truck leg to push against an obstacle, force feedback let the driver feel the obstacle as though it were his or her own arm or leg doing the pushing.

After about 20 hours of training, Mosher was able to handle the machine with surprising agility. Films of the machine operating under his control show it ambling along at about 5 MPH, climbing a stack of railroad ties, pushing a foundered jeep out of the mud, and maneuvering a large drum onto some hooks. Despite its dependence on a well-trained human for control, this walking machine was a landmark in legged technology.



$$CD = AD = DM = \frac{3 + \sqrt{7}}{2}$$

$$BC = \frac{4 + \sqrt{7}}{3}$$

When the input crank *AB* rotates, the output point *M* moves along a straight path during one part of the cycle and an arched path during the other part. Two identical linkages are arranged to operate out of phase so at least one provides a straight motion at all times. The body is always supported by feet connected to the straight-moving linkage. Linkages of this sort, consisting of pivots and rigid members, are a simple means of generating patterned motion. After Lucas [13].

**FIGURE 1. Linkage Used in an Early Walking Machine**

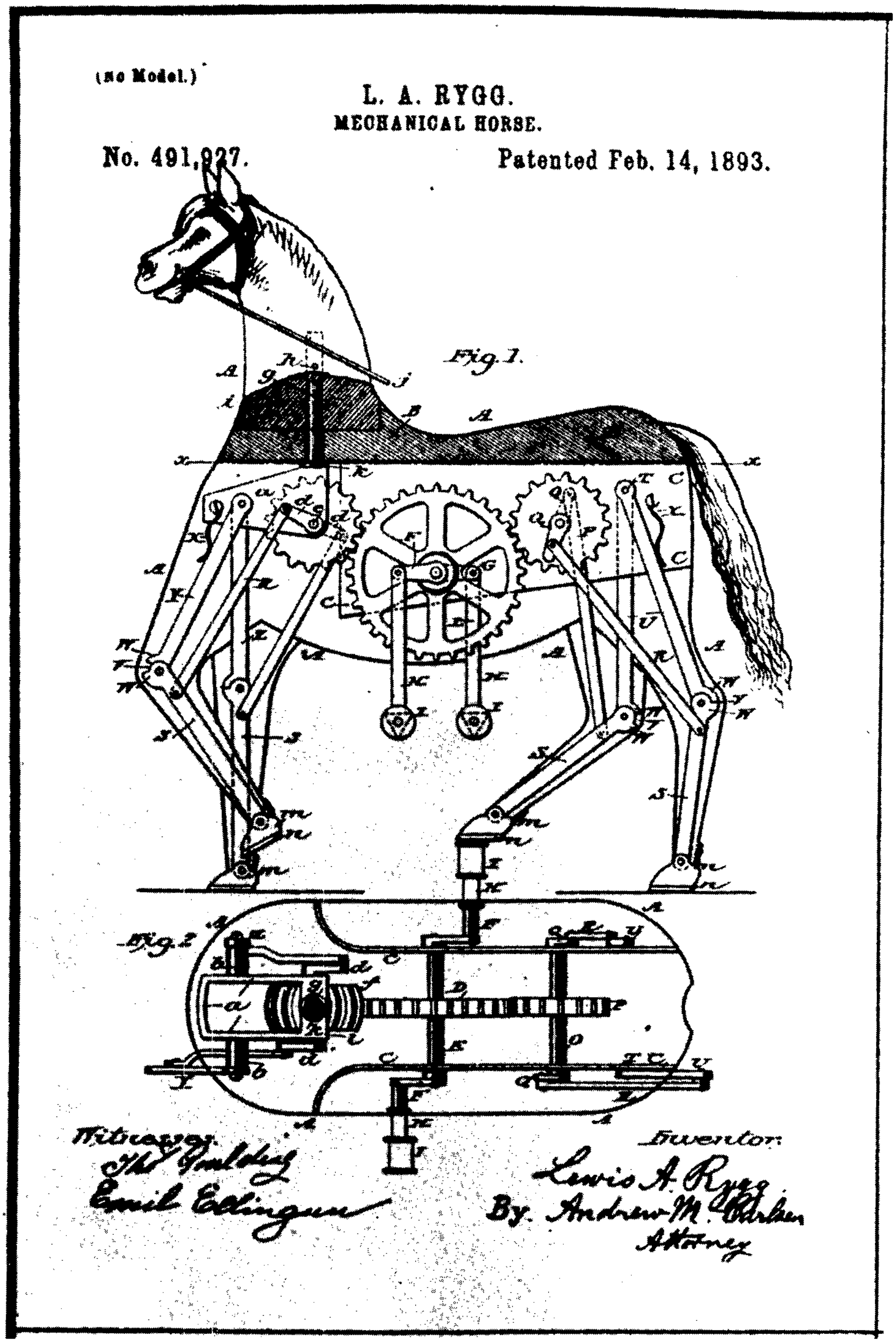
Computer control became an alternative to human control of legged vehicles in the 1970s. Robert McGhee's group at the Ohio State University was the first to use this approach successfully [16]. In 1977 they built an insectlike hexapod that could walk with a number of standard gaits, turn, walk sideways, and negotiate simple obstacles. The computer's primary task was to solve kinematic equations in order to coordinate the 18 electric motors driving the legs. This coordination ensured that the machine's center of mass stayed over the polygon of support provided by the feet while allowing the legs to sequence through a gait (Figure 4, page 504). The machine traveled quite slowly, covering several yards per minute. Force and visual sensing provided a measure of terrain accommodation in later developments.

The hexapod provided McGhee with an excellent opportunity to pursue his earlier theoretical findings on the combinatorics and selection of gait [10, 15, 18]. The group at Ohio State is currently building a much larger hexapod (about 3 tons), which is intended to operate on rough terrain with a high degree of autonomy [40].

Gurfinkel and his co-workers in the USSR built a machine with characteristics and performance quite similar to McGhee's at about the same time [3]. It used a hybrid computer for control, with heavy use of analog computation for low-level functions.

Hirose realized that linkage design and computer control were not mutually exclusive. His experience with clever and unusual mechanisms—he had built seven kinds of mechanical snakes—led to a special leg that simplified the control of locomotion and could improve efficiency [6, 7]. The leg was a three-dimensional pantograph that translated the motion of each actuator into a pure Cartesian translation of the foot. With the ability to generate $x$, $y$, and $z$ translations of each foot by merely choosing an actuator, the control computer was freed from the arduous task of performing kinematic solutions. The mechanical linkage was actually helping to perform the calculations needed for locomotion. The linkage was efficient because the actuators performed only positive work in moving the body forward.

Hirose used this leg design to build a small quadruped, about one yard long. It was equipped with touch sensors on each foot and an oil-damped pendulum attached to the body. Simple algorithms used



This device was patented by Lewis A. Rygg in 1893. The stirrups double as pedals so the rider can power the stepping motions. The reins move the head and forelegs from side to side for steering. Apparently this machine was never built.

**FIGURE 2. Mechanical Horse**

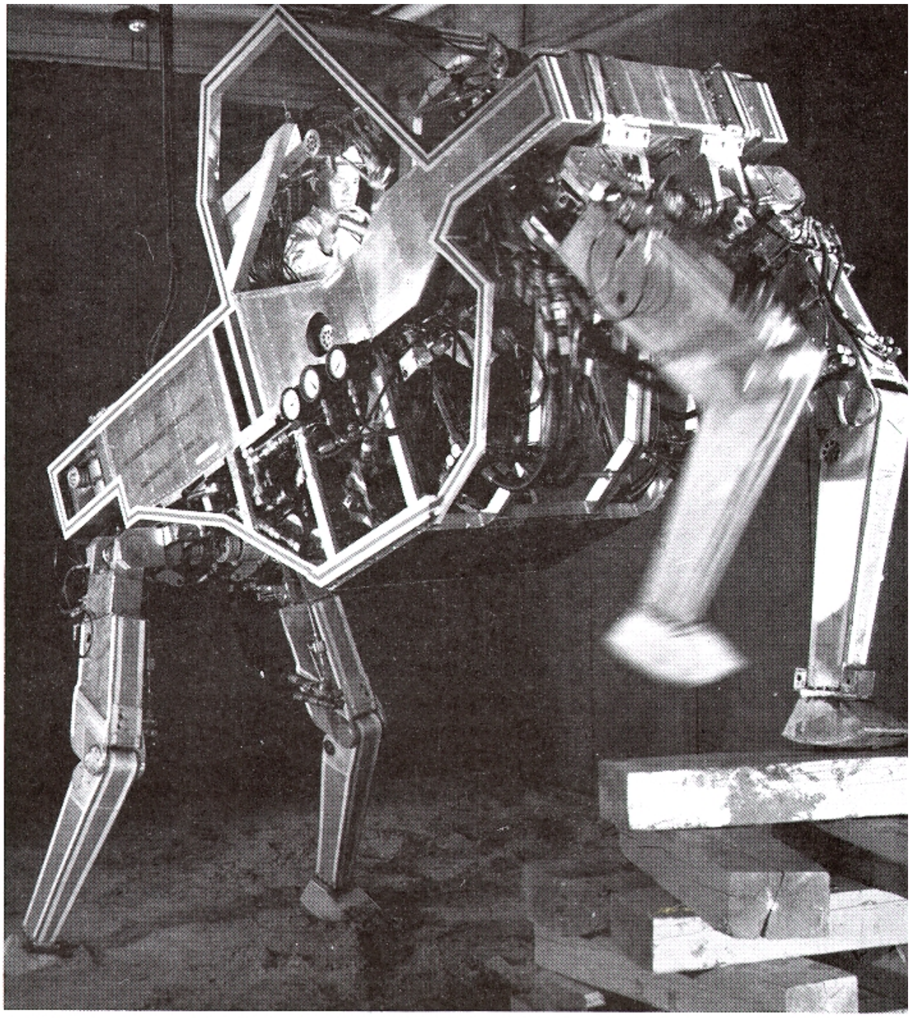
the sensors to control the actions of the feet. For instance, if a touch sensor indicated contact while the foot was moving forward, the leg would move backward a little bit, move upward a little bit, then resume its forward motion. If the foot had not cleared the obstacle, the cycle would repeat. The use of several simple algorithms like this one permitted Hirose's machine to climb up and down stairs and to negotiate other obstacles without human intervention [6].

These three walking machines, McGhee's, Gurfinkel's, and Hirose's, represent a class called *static crawlers*. Each differs in the details of construction and in the computing technology used for control, but shares a common approach to balance and stability. Enough feet are kept on the ground to guarantee a broad base of support at all times, and the body and legs move to keep the center of mass over this broad support base. The forward velocity is kept sufficiently low so that stored energy need not be figured into the stability calculation. Each of these machines has been used to study rough terrain locomotion in the laboratory through experiments on terrain sensing, gait selection, and selection of foothold sequences. Several other machines that fall into this class have been studied in the intervening years, for example, see [31] and [37].

## DYNAMICS AND BALANCE IMPROVE MOBILITY

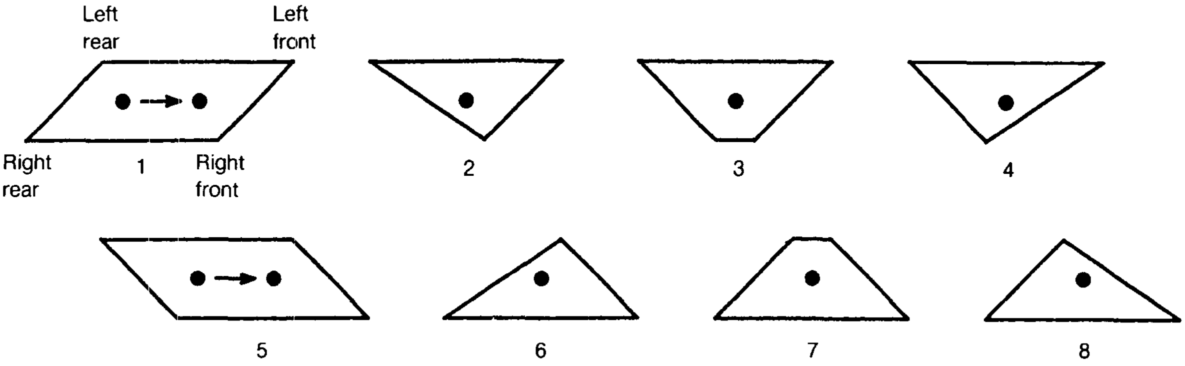
We now consider the study of dynamic machines that balance actively. This means that the legged systems studied operate in a regime where the velocities and kinetic energies of the masses are important determinants of behavior. In order to predict and influence the behavior of a dynamic system, we must consider the energy stored in each mass and spring as well as the geometric structure and configuration of the mechanism. Geometry and configuration taken alone do not provide an adequate model when a system moves with substantial speed or has large mass. Consider, for example, a fast-moving vehicle with its center of mass too close to the front feet: It would tip over if it stopped suddenly.

The exchange of energy among its various forms is



This vehicle was developed by Ralph Mosher at General Electric in about 1968. The human driver controlled the machine with four handles and pedals that were hydraulically connected to the four legs. Photograph courtesy of General Electric Research and Development Center.

FIGURE 3. Walking Truck



The diagram shows the sequence of support patterns provided by the feet of a quadruped walking with a crawling gait. The body and legs move to keep the projection of the center of mass within the polygon defined by the feet. A supporting foot is located at each vertex. The dot indicates the projection of the center of mass. Adapted from McGhee and Frank [17].

**FIGURE 4. Statically Stable Gait**

also important in the dynamics of legged locomotion. For example, there is a cycle of activity in running that changes the form of the stored energy several times: The body's potential energy of elevation changes into kinetic energy during falling, then into strain energy when parts of the leg deform elastically during rebound with the ground, then into kinetic energy again as the body accelerates upward, and finally back into potential energy of elevation. This sort of dynamic exchange is central to an understanding of legged locomotion.

Dynamics also plays a role in giving legged systems the ability to balance actively. A statically balanced system avoids tipping and the ensuing horizontal accelerations by keeping its center of mass over the polygon of support formed by the feet. Animals sometimes use this sort of balance when they move slowly, but they usually balance actively.

A legged system that balances actively can tolerate departures from static equilibrium. Unlike a statically balanced system, which must always operate in or near equilibrium, an actively balanced system is permitted to tip and accelerate for short periods of time. The control system manipulates body and leg motions to ensure that each tipping interval is brief and that each tipping motion in one direction is compensated by a tipping motion in the opposite direction. An effective base of support is thus maintained over time. A system that balances actively can also tolerate vertical acceleration, such as the ballistic flight and bouncing that occur during running.

Cannon and his students built machines that balanced inverted pendulums on a moving cart. They balanced two pendulums side by side, one pendulum on top of another, and a long limber inverted pendulum. Only one input, the force driving the cart horizontally, was available for control. Adapted from Schaefer and Cannon [32].

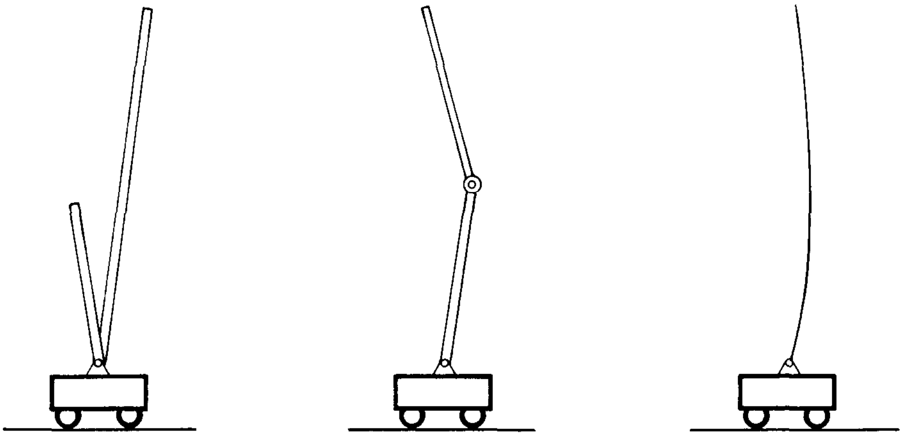


**FIGURE 5. Balancing Inverted Pendulums**

This machine is shown traveling at about 1.75 MPH from right to left. Lines made by light sources attached to the machine indicate paths of the foot and the hip.

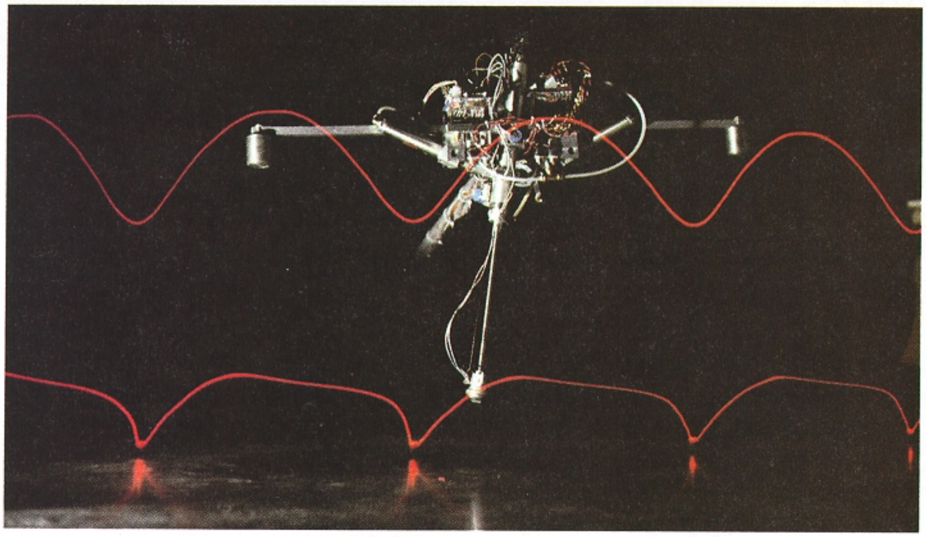


FIGURE 6. Planar Hopping Machine

The control system operates to regulate hopping height, forward velocity, and body posture. Top recorded running speed was about 4.8 MPH.

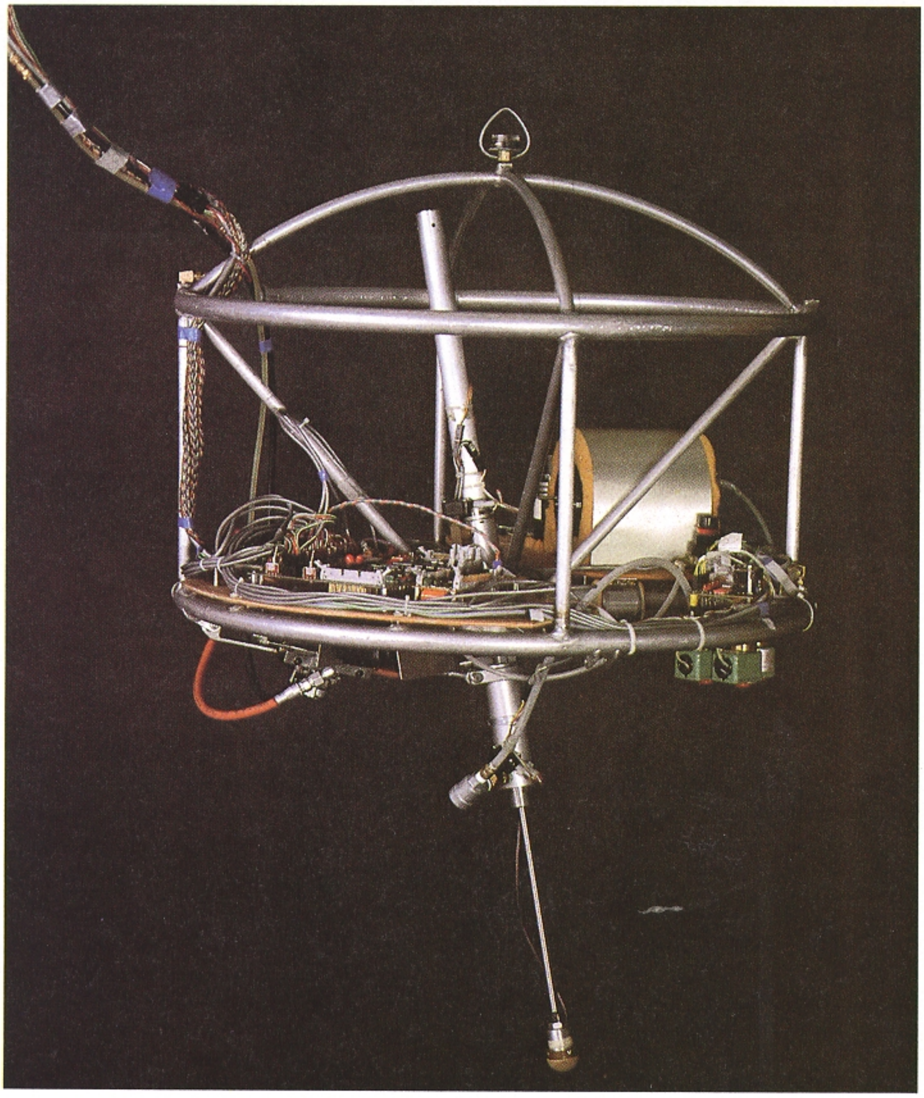
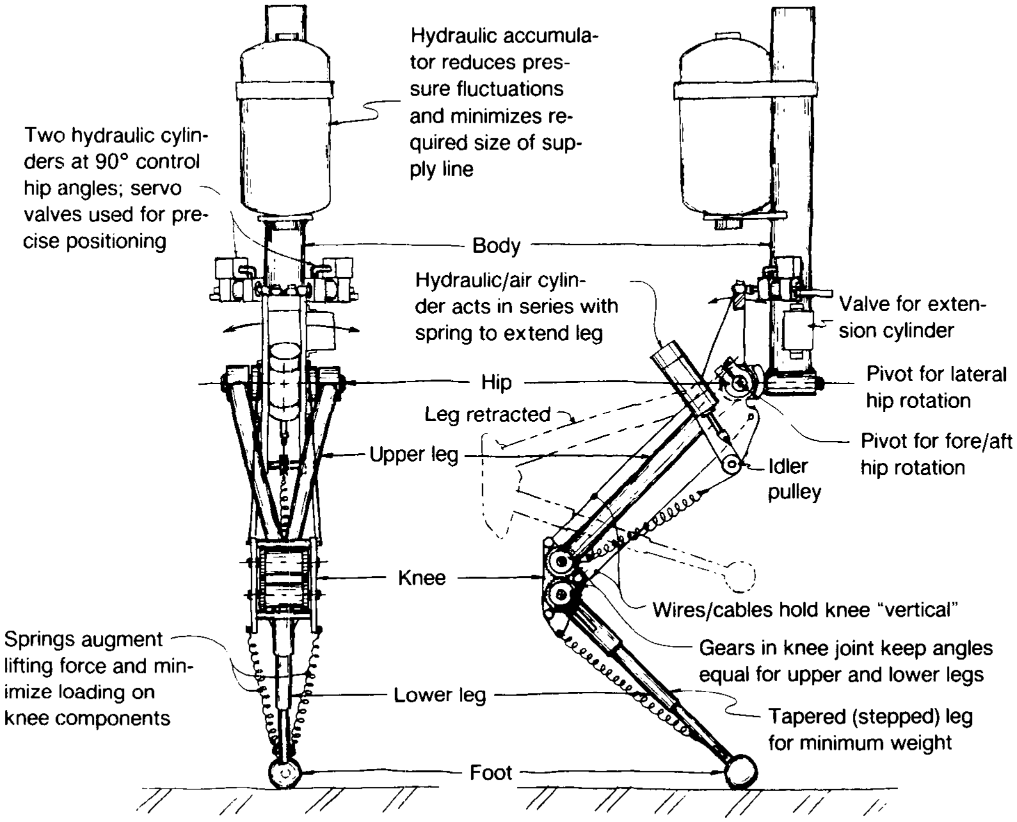


FIGURE 7. Three-Dimensional Hopping Machine



Ben Brown and I had this early concept for a one-legged hopping machine that was to operate in three dimensions. The design never left the drawing board.

**FIGURE 8. Design for Three-Dimensional Hopping Machine**

The ability of an actively balanced system to depart from static equilibrium relaxes the rules governing how legs can be used for support, which in turn leads to improved mobility. For example, if a legged system can tolerate tipping, then it can position its feet far from the center of mass in order to use widely separated or erratically placed footholds. If it can remain upright with a small base of support, then it can travel where obstructions are closely spaced or where the path of firm support is narrow. The ability to tolerate intermittent support also contributes to mobility by allowing a system to move all its legs to new footholds at one time, to jump onto or over obstacles, and to use short periods of ballistic flight for increased speed. These abilities to use narrow-base and intermittent support generally increase the types of terrain a legged system can negotiate. Animals routinely exploit active balance to travel quickly on difficult terrain; legged vehicles will have to balance actively, too, if they are to move with animallike mobility and speed.

## RESEARCH ON ACTIVE BALANCE

The first machines that balanced actively were automatically controlled inverted pendulums. Everyone knows that a human can balance a broom on a finger with relative ease. Why not use automatic control to build a broom that can balance itself? Claude Shannon was probably the first to do so. In 1951 he used the parts from an erector set to build a machine that balanced an inverted pendulum atop a small powered truck. The truck drove back and forth in response to the tipping movements of the pendulum, as sensed by a pair of switches at its base. In order to move from one place to another, the truck first had to drive away from the goal to unbalance the pendulum toward the goal; in order to balance again at the destination, the truck moved past the destination until the pendulum was again upright with no forward velocity, then moved back to the goal.

At Shannon's urging, Robert Cannon and two of his students at Stanford University set about demonstrating controllers that balanced two pendulums at once. In one case, the pendulums were mounted side by side on the cart, and in the other, they were mounted one on top of the other (Figure 5, page 504). Cannon's group was interested in the single-input multiple-output problem and in the limitations of achievable balance: How could they use the single force that drove the cart's motion to control the angles of two pendulums as well as the position of the cart? How far from balance could the system deviate before it was impossible to return to equilibrium, given such parameters of the mechanical system as the cart motor's strength and the pendulum's length?

Using analysis based on normal coordinates and optimal switching curves, Cannon's group expressed regions of controllability as explicit functions of the physical parameters of the system. Once these regions were found, their boundaries were used to find switching functions that provided control [5]. Later, they extended these techniques to provide balance for a flexible inverted pendulum [32]. These studies of balance for inverted pendulums were important precursors to later work on locomotion. The inverted-pendulum model for walking would become the primary tool for studying balance in legged systems [4, 9, 20, 39]. It is unfortunate that no one has yet extended Cannon's elegant analysis to the more complicated legged case.

Mosher's group at General Electric was also interested in balance. Their original intention, before deciding to build the quadruped described earlier, was to build a walking biped that would be controlled by a human who would "walk" in an instrumented harness inside the cockpit. They started with a human-factors experiment because they were unsure of the human's ability to adjust to the exaggerated vestibular input that would be experienced in driving a legged machine several times taller than the driver. In the experiments, the subjects stood on an inverted pendulum about 20 feet tall. The pendulum had pivots like an ankle and hip, one at the floor and one just below the platform that supported the subject. These pivots were controlled to follow the corresponding ankle and hip motions of the subject. Each of the 86 people tested learned to balance the machine in less than 15 minutes, and most learned in just 2 or 3 minutes [12]. Although the GE walking truck mentioned earlier could, in principle, operate using purely static techniques, the driver's ability to balance it actively probably contributed to its smooth operation. A GE walking biped was never built.

The importance of active balance in legged loco-

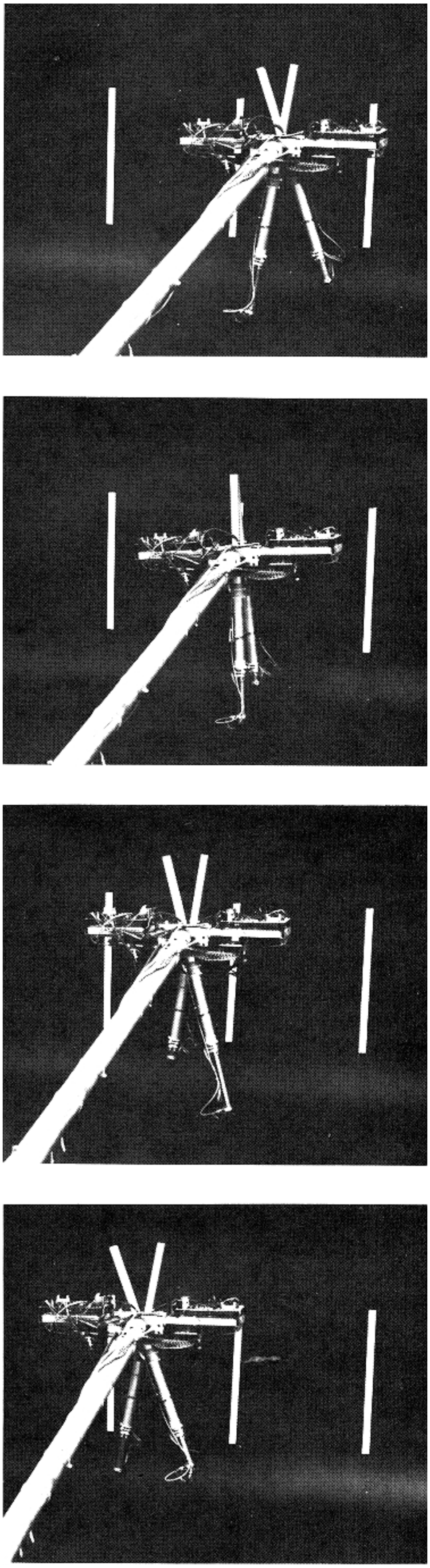


**FIGURE 9. Planar Biped**

This machine can run with a gait that uses the legs in alternation like a human, or with a hopping gait, and it can switch back and forth between gaits. The two legs counteroscillate during normal running. Top running speed was 9.5 MPH. The control is based on the three-part decomposition originally used for the one-legged hopping machines. During one-legged hopping, the extra leg acts like a tail, swinging out of phase with the active leg. From Hodgins, Koechling, and Raibert [8].

motion had been widely recognized for some years [2, 19, 39], but progress in building physical legged systems that employ such principles was retarded by the perceived difficulty of the task. It was not until the late 1970s that experimental work on balance in legged systems got under way.

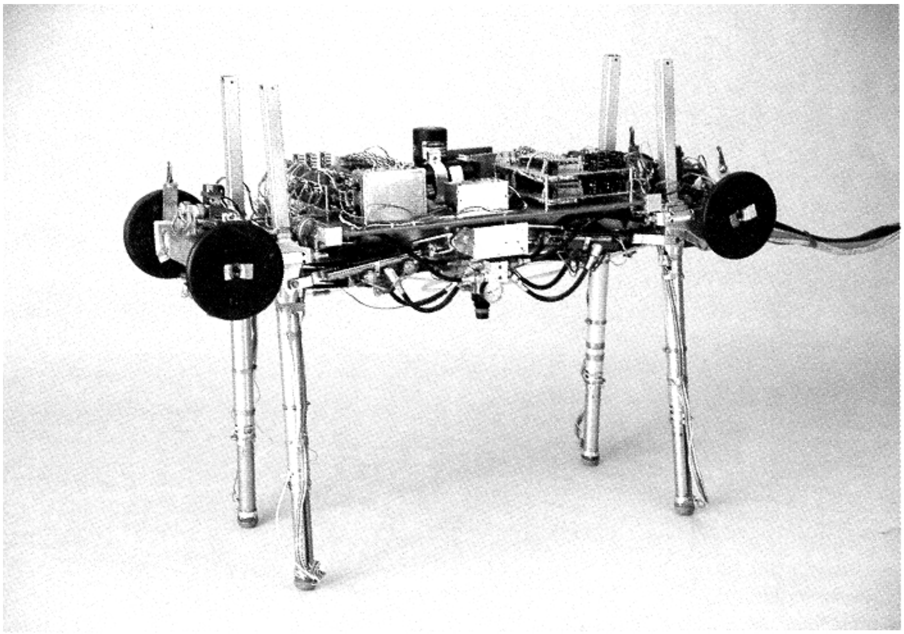
Kato and his co-workers built a biped that walked with a *quasi-dynamic* gait [9]. The machine had 10 hydraulically powered degrees of freedom and two large feet. This machine was usually a static crawler, moving along a preplanned trajectory to keep the center of mass over the base of support provided by the supporting foot. Once during each step, however, the machine temporarily destabilized itself to tip forward so that support would be transferred quickly from one foot to the other. Before the transfer took place on each step, the *catching* foot was positioned to return the machine to equilibrium passively. No active response was required. A modified inverted-pendulum model was used to plan the tipping motion.

In 1984 this machine walked with a quasi-dynamic gait, taking about a dozen 0.4-m-long steps per minute. The use of a dynamic transfer phase makes an important point: A legged system can employ complicated dynamic behavior without requiring a very complicated control system.

Miura and Shimoyama [20] built the first walking machine that really balanced actively. Their stilt biped was patterned after a human walking on stilts. Each foot provided only a point of support, and the machine had three actuators: one for each leg that moved the leg sideways and a third that separated the legs fore and aft. Because the legs did not change length, the hips were used to pick up the feet. This gave the machine a pronounced shuffling gait reminiscent of Charlie Chaplin's stiff-kneed walk.

Control for the stilt biped relied, once again, on an inverted-pendulum model of its behavior. Each time a foot was placed on the floor, its position was chosen according to the tipping behavior expected from an inverted pendulum. Actually, the problem was broken down as though there were two pendulums, one in the pitching plane and one in the rolling plane. The choice of foot position along each axis took the current and desired state of the system into account. The control system used tabulated descriptions of planned leg motions together with linear feedback to perform the necessary calculations. Unlike Kato's machine, which came to static equilibrium before and after each dynamic transfer, the stilt biped tipped all the time.

Matsuoka [14] was the first to build a machine that ran, where running is defined by the presence of intervals of ballistic flight when all feet are off the ground. Matsuoka's goal was to model repetitive hopping in humans. He formulated a model with a body and one massless leg, and he simplified the



The control system for this multilegged machine coordinates more than one leg to behave as a single "virtual" leg. This reduces four-legged running to biped running, which can be controlled with the one-leg algorithm.

FIGURE 10. Quadruped Machine

problem by assuming that the duration of the support phase was short compared with the ballistic-flight phase. This extreme form of running, in which nearly the entire cycle is spent in flight, minimizes the influence of tipping during support. This model permitted Matsuoka to derive a time-optimal state feedback controller that provided stability for hopping in place and for low-speed translations.

To test his method for control, Matsuoka built a planar one-legged hopping machine. The machine operated at low gravity by rolling on ball bearings on a table that was inclined 10 degrees from the horizontal. An electric solenoid provided a rapid thrust at the foot, so the support period was short. The machine hopped in place at about one hop per second and traveled back and forth on the table.

## RUNNING MACHINES

Running is a form of legged locomotion that uses ballistic-flight phases to obtain high speed. To study running, my co-workers and I have explored a variety of legged systems and implemented some of them in the form of physical machines. To study running in its simplest form, we built a machine that ran on just one leg (see Figure 6, page 505). The machine hopped like a kangaroo, using a series of leaps. A machine with only one leg allowed us to concentrate on active balance and dynamics while postponing the difficult task of coordinating the behavior of many legs. Gait has been a dominant issue in legged locomotion for some years, but we wonder how central it really is. Are there algorithms for walking and running that are independent of gait or that work correctly for any number of legs? Perhaps a machine with just one gait could suggest answers to these questions.

The first machine we built to study these problems had two main parts: a body and a leg [28]. The body carried the actuators and instrumentation needed for the machine's operation. The leg could telescope to change length, was springy along the telescoping axis, and could pivot with respect to the body at a simple hip. Sensors measured the pitch angle of the body, the angle of the hip, the length of the leg, the tension in the leg spring, and contact with the ground. This first machine was constrained to operate in a plane, so it could move only up and down and fore and aft and rotate in the plane. An umbilical cable connected the machine to power and to a control computer.

For this machine, running and hopping are the same. The running cycle has two phases. During one phase, called *stance*, the leg supports the weight of the body, and the foot stays in a fixed location on the ground. During stance, the system tips like an inverted pendulum. During the other phase, called *flight*, the center of mass moves ballistically, with the leg unloaded and free to move.

### Control of Running Was Decomposed into Three Parts

We were surprised to find that a simple set of algorithms could control this planar one-legged hopping machine. Our approach was to consider the hopping motion, forward travel, and posture of the body separately. This decomposition led to a control system with three parts:

- *Hopping.* One part of the control system excited the cyclic hopping motion that underlies running while regulating the height to which the machine hopped. The hopping motion is an oscillation governed by the mass of the body, the springiness of the leg, and gravity. During stance, the body bounced on the springy leg, and during flight, the system traveled a ballistic trajectory. The control system delivered a vertical thrust with the leg during each support period to sustain the oscillation and to regulate its amplitude. Some of the energy needed for each hop was recovered by the leg spring from the previous hop.
- *Forward speed.* A second part of the control system regulated the forward running speed and acceleration. This was done by moving the leg to an appropriate forward position with respect to the body during the flight portion of each cycle. The position of the foot with respect to the body when landing has a strong influence on the behavior during the support period that follows. Depending on where the control system places the foot, the body will either continue to travel with the same forward speed, accelerate to go faster, or slow down. To calculate a suitable forward position for the foot, the control system takes account of the actual forward speed, the desired speed, and a simple model of the legged system's dynamics. A single algorithm works correctly when the machine is hopping in place, accelerating to a run, running at a constant speed, and slowing to a stationary hop.
- *Posture.* The third part of the control system stabilizes the pitch angle of the body to keep the body upright. Torques exerted between the body and leg about the hip accelerate the body about its pitch axis, provided that there is good traction between the foot and the ground. During the support period, there is traction because the leg supports the load of the body. Linear feedback control oper-

ates on the hip actuator during each support period to restore the body to an upright posture.

Breaking running down into the control of these three functions was important for simplifying the problem. Each part of the control system acted as though it influenced just one component of the behavior, and the interactions that resulted from imperfect decoupling were treated as disturbances. Partitioning the control into these three parts made running much easier to understand and led to a fairly simple control system. The algorithms implemented to perform each part of the control task were themselves simple, although none was optimized for performance. The details of the individual control algorithms are not so important as the framework provided by the decomposition.

Using the three-part control system, the planar one-legged machine hopped in place, traveled at a specified rate, maintained balance when disturbed, and jumped over small obstacles. Top running speed was about 2.6 MPH. The utility of the decomposition and framework was not limited to planar hopping on one leg—the approach was generalized for controlling a three-dimensional one-legged machine, a planar two-legged machine, and a quadruped.

## Locomotion in Three Dimensions

The one-legged machine was constrained mechanically to operate in the plane, but useful legged systems must balance themselves in three-dimensional space. Can the control algorithms used for hopping in the plane be generalized somehow for hopping in three dimensions? A key to answering this question was the recognition that animal locomotion is primarily a planar activity, even though animals are three-dimensional systems. Films of a kangaroo hopping on a treadmill first suggested this point. The legs sweep fore and aft through large angles with the tail sweeping in counteroscillation with the legs, and the body bouncing up and down. These motions all occur in the sagittal plane, with little or no motion normal to the plane.

Sesh Murthy realized that the plane in which all this activity occurs can generally be defined by the forward velocity vector and the gravity vector. He called this the *plane of motion* [23]. For a legged system without a preferred direction of travel, the plane of motion might vary from stride to stride, but it would be defined in the same way. We found that the three-part control system retained its effectiveness when used to control activity within the plane of motion.

We also found, however, that the mechanisms needed to control the remaining *extraplanar* degrees of freedom could be cast in a form that fit into the original three-part framework. For instance, the algorithm for placing the foot to control forward speed became a vector calculation. One component of foot placement determined forward speed in the plane of motion, whereas the other component caused the plane to rotate about a vertical axis, permitting the control system to steer. A similar extension applied to body posture. The result was a three-part control system for three dimensions that was derived directly from the one used for the planar case.

To explore these ideas, we built the design shown in Figure 7 (page 505), and designed further hopping machines (Figure 8, page 506). The machine in Figure 7 had an additional joint at the hip to permit the leg to move sideways as well as fore and aft, and had no external mechanical support. Otherwise, it was similar to the planar hopping machine. In operation, this machine balanced itself as it hopped along simple paths in the laboratory, traveling at a top speed of 4.8 MPH [29].

## Running on Several Legs

Experiments on machines with one leg were not motivated by an interest in one-legged vehicles. Although such vehicles might very well turn out to have merit,[1] our interest was in getting at the basics of active balance and dynamics in the context of a simplified locomotion problem. In principle, results from machines with one leg could have value for understanding all sorts of legged systems, perhaps with any number of legs.

Given the successful control of machines that run and balance on one leg, can we use what was learned to understand and control machines with several legs? Our study of this problem has progressed in two stages. For a biped that runs like a human, with alternating periods of support and flight, the one-leg control algorithms apply directly. Because the legs are used in alternation, only one leg is active at a time: One leg is placed on the ground at a time, one leg thrusts on the ground at a time, and one leg can exert a torque on the body at a time. We call this sort of running a *one-foot gait.* Assuming that the behavior of the other leg does not interfere, the one-leg algorithms for hopping, forward travel, and posture can each be used to control the active leg. Of course, to make this workable, some bookkeeping is required to keep track of which leg is active and to keep the extra leg out of the way.

---

[1] Wallace and Seifert saw merit in vehicles with one leg. Wallace [41] patented a one-legged hopping tank that was supposed to be hard to hit because of its erratic movements. Seifert [33] proposed the *lunar pogo* as a means of efficient travel on the moon.

Jessica Hodgins and Jeff Koechling demonstrated the effectiveness of this approach by using the one-leg algorithms to control each leg of a planar biped [8]. The machine, shown in Figure 9 (page 507), has run at 9.5 MPH. In addition to running with an alternating gait, the biped can run by hopping on one leg, and can switch back and forth between gaits. We found that it was very simple to extend the one-leg algorithms for two-legged running.

In principle, this approach could be used to control any number of legs, so long as just one is active at a time. Unfortunately, when there are several legs this is usually not feasible. Suppose, however, that a control mechanism coordinated groups of legs that shared support simultaneously, making them behave like a single equivalent leg—what Ivan Sutherland [37] has called a *virtual leg.* Suppose further that more than one leg at a time provides support, but that all support legs are coordinated to act like a virtual leg. Several multilegged gaits can then be mapped into *virtual biped one-foot gaits.* For example, the trotting quadruped maps into a virtual biped running with a one-foot gait.

We argue that the trotting quadruped is like a biped, that a biped is like a one-legged machine, and that control of one-legged machines is a solved problem. A control system for quadruped trotting could consist of a controller that coordinates each pair of legs to act like one virtual leg, a three-part control system that acts on the virtual legs, and a bookkeeping mechanism. Figure 10, page 508, shows a four-legged machine that has run with precisely this sort of control system [30]. Table II and [26] summarize work on running machines.

**TABLE II. Summary of Research in the CMU Leg Laboratory**

| | |
|---|---|
| 1982 | Planar one-legged machine hops in place, travels at specified rate of up to 2.6 MPH, tolerates mechanical disturbances, and jumps over small obstacles. |
| 1983 | One-legged hopping machine runs on open floor, balancing in three dimensions. Top speed is about 4.5 MPH. |
| 1983 | Murphy finds passively stabilized bounding gait for simulated quadrupedlike model. |
| 1984 | Cat and human are found to run with symmetry like running machines. |
| 1984 | Quadruped runs with trotting gait. Virtual legs permit use of one-leg control algorithms. |
| 1985 | Planar biped runs with one- and two-legged gaits and can change gait while running. Top speed is 9.5 MPH. |

## Computer Programs for Running

The behavior of the running machines just described was controlled by a set of computer programs that ran on our laboratory computer. These computer programs performed several functions including

- sampling and filtering data from the sensors;
- transforming kinematic data between coordinate systems;
- executing the three-part locomotion algorithms for hopping, forward speed, and body attitude;
- controlling the actuators;
- reading operator instructions from the console;
- recording running behavior.

The control computer was a VAX-11/780 running the UNIX® operating system.[2] In order to provide real-time service with short latency and high bandwidth feedback, the real-time control programs were implemented as a device driver that resided within the UNIX kernel. The device driver responded with short latency to a hardware clock that interrupted through the UNIBUS every 8 ms. All sensors and actuators were also accessed through interfaces that connected to the UNIBUS. Each time the clock ticked, the running machine driver programs sampled and scaled data from the sensors, estimated joint and body velocities to determine the state of the running machine, executed the three-part locomotion algorithms, and calculated a new output for each actuator.

The programs performing these tasks were written in a mixture of C and assembly language. Assembly language was used where speed was of primary concern, such as in performing the kinematic transformations used to convert between coordinate systems. In some cases, tabulated data were used to further increase the speed of a transformation. For instance, trigonometric functions, square roots, and higher order kinematic operations were evaluated with linear interpolation among precomputed tabulated data. One such evaluation takes about 15 $\mu$s. The kinematic relationship between the quadruped's fore/aft hip actuator length and the forward position of the foot in body coordinates exemplifies a function that was evaluated with a table.

The control programs were synchronized to the behavior of the running machine by a software finite state machine. The state machine made transitions from one state to another when sensory data from the running machine satisfied the specified conditions. For instance, the state machine made a transi-

---

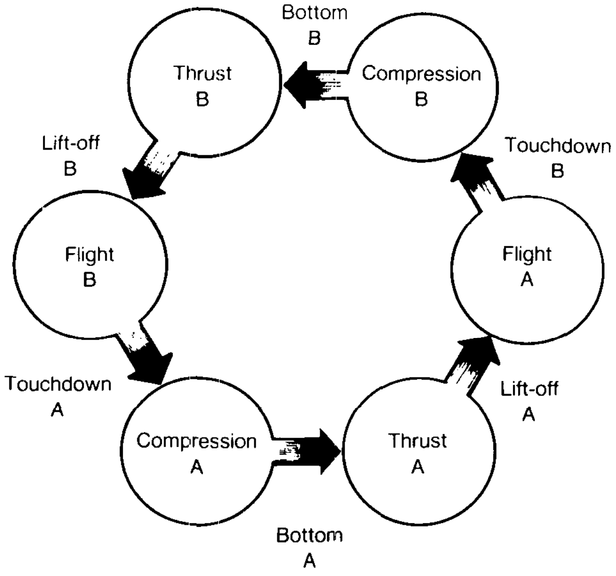
UNIX is a trademark of AT&T Bell Laboratories.

[2] The planar one-legged machine was controlled by a PDP-11/40 programmed in assembly language.

tion from COMPRESSION to THRUST when the derivative of the support leg's length changed from negative to positive. Figure 11 and Table III give some detail for the state machine that was used for the biped and quadruped running machines. We found that using a state machine along with properly designed transition conditions aided the interpretation of sensory data by providing noise immunity and hysteresis.

Whereas sensory data determine when the state machine makes transitions, the resulting states determine which control algorithms operate to provide control. For instance, when the biped is in the THRUST A state, the control programs extend leg A, exert torque on hip A, shorten leg B, and position foot B.

In addition to the real-time programs that control the running machines, a top-level program was used to control the real-time programs. The top-level program permitted the user to initiate a running experiment, select among control modes, examine or modify the variables and parameters used by the control programs, specify sensor calibration data, mark variables for recording, and save recorded real-time data for later analysis and debugging. Each of these functions was accomplished by one or more system calls to the driver. The top-level program had no particular time constraints, so it was implemented as a time-sharing job that was scheduled by the normal UNIX scheduler.



This is a simplified diagram of the state machine that synchronizes the control programs to the behavior of the running machine. This diagram is for the biped and quadruped state machines, but the one-legged state machines are similar. State transitions are determined by sensory events related to the hopping motion. A different set of control actions is put into effect in each state, as indicated in Table III.

**FIGURE 11. State Machine that Controls Running**

**TABLE III. Details of the State Machine for Biped and Quadruped**

| State | Trigger event | Action |
|---|---|---|
| 1 LOADING A | A touches ground | Zero hip torque A<br>Shorten B<br>Do not move hip B |
| 2 COMPRESSION A | A air spring shortened | Erect body with hip A<br>Shorten B<br>Position B for landing |
| 3 THRUST A | A air springs lengthening | Extend A<br>Erect body with hip A<br>Keep B short<br>Position B for landing |
| 4 UNLOADING A | A air spring near full length | Shorten A<br>Zero hip torque A<br>Keep B short<br>Position B for landing |
| 5 FLIGHT A | A not touching ground | Shorten A<br>Do not move hip A<br>Lengthen B for landing<br>Position B for landing |

States 6–10 repeat states 1–5, with A and B interchanged.

The state shown in the left column is entered when the event listed in the center column occurs. States advance sequentially during normal running. During states 1–5, leg A is in support, and leg B is in recovery. During states 6–10, these roles are reversed. For biped running, A refers to leg 1, and B refers to leg 2. For quadruped trotting, each letter designates a pair of physical legs.

Animals are shown in symmetric configuration halfway through the stance phase for several gaits: rotary gallop (top), transverse gallop (second), canter (third), and amble (bottom). In each case, the body is at minimum altitude, the center of support is located below the center of mass, the rearmost leg was recently lifted, and the frontmost leg is about to be placed. Photographs from Muybridge [25]; reprinted with permission from Dover Press.

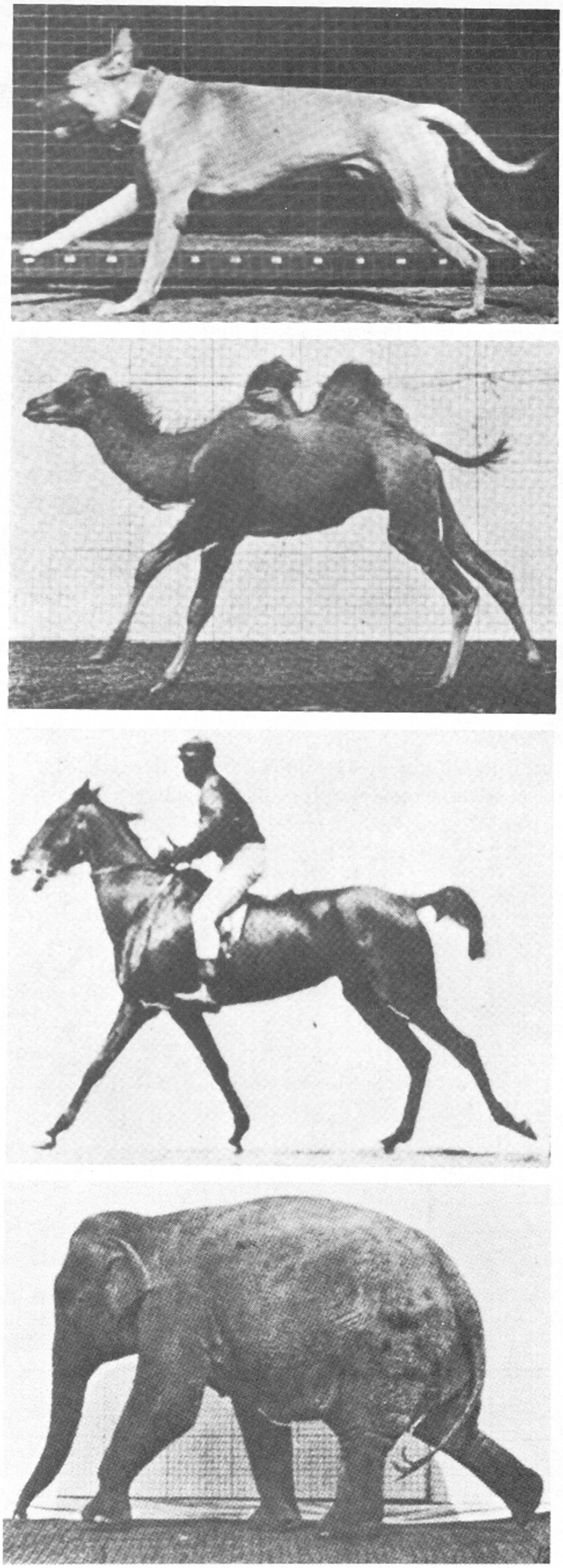


**FIGURE 12. Symmetry in Animal Locomotion**

## Symmetry in Robots and Animals

In order to run at constant forward speed, the instantaneous forward accelerations that occur during a stride must integrate to zero. One way to satisfy this requirement is to organize running behavior so that forward acceleration has an odd symmetry throughout each stride—functions with odd symmetry integrate to zero over symmetric limits.[3] This sort of symmetry was used to control forward speed in all four machines just described. It was accomplished by choosing an appropriate forward position for the foot on each step. In principle, symmetry of this sort can be used to simplify locomotion in systems with any number of legs and for a wide range of gaits.

Can the symmetries developed for legged machines help us to understand the behavior of legged animals? To find out, we have examined film data for running animals and humans (see Figure 12, page 513). In particular, we have looked at a cat trotting and galloping on a treadmill and a human running on a track [27]. The data conform reasonably well to the predicted even and odd symmetries. In some cases, the data are remarkably symmetric.

***Acknowledgments.*** This research was supported by a contract from the Engineering Applications Office of the Defense Advanced Research Projects Agency and a grant from the System Development Foundation.

---

[3] If $x(t)$ is an odd function of time, then $x(t) = -x(-t)$. If $x(t)$ is even, then $x(t) = x(-t)$.

**REFERENCES**

Reference [22] is not cited in text.

1. Ehrlich, A. Vehicle propelled by steppers. U.S. Patent 1,691,233, Adolf Ehrlich, Budapest, Hungary, 1928.
2. Frank, A.A. An approach to the dynamic analysis and synthesis of biped locomotion machines. *Med. Biol. Eng. 8* (1970), 465–476.
3. Gurfinkel, V.S., Gurfinkel, E.V., Shneider, A.Y., Devjanin, E.A., Lensky, A.V., and Shitilman, L.G. Walking robot with supervisory control. *Mech. Mach. Theory 16* (1981), 31–36.
4. Hemami, H., and Golliday, C.L., Jr. The inverted pendulum and biped stability. *Math. Biosci. 34* (1977), 95–110.
5. Higdon, D.T., and Cannon, R.H., Jr. On the control of unstable multiple-output mechanical systems. In *ASME Winter Annual Meeting*. 1963.
6. Hirose, S. A study of design and control of a quadruped walking vehicle. *Int. J. Robotics Res.* 3 (1984), 113–133.
7. Hirose, S., and Umetani, Y. The basic motion regulation system for a quadruped walking vehicle. In *ASME Conference on Mechanisms* (Los Angeles, Calif.). 1980.
8. Hodgins, J., Koechling, J., and Raibert, M.H. Running experiments with a planar biped. In *3rd International Symposium on Robotics Research*, G. Giralt and M. Ghallab, Eds. MIT Press, Cambridge, Mass., 1985.
9. Kato, T., Takanishi, A., Jishikawa, H., and Kato, I. The realization of the quasi-dynamic walking by the biped walking machine. In *4th Symposium on Theory and Practice of Robots and Manipulators*, A. Morecki, G. Bianchi, and K. Kedzior, Eds. Polish Scientific Publishers, Warsaw, Poland, 1983, pp. 341–351.
10. Koozekanani, S.H., and McGhee, R.B. Occupancy problems with pairwise exclusion constraints—An aspect of gait enumeration. *J. Cybern.* 2 (1973), 14–26.
11. Liston, R.A. Increasing vehicle agility by legs: The quadruped transporter. In *38th National Meeting of the Operations Research Society of America*. 1970.
12. Liston, R.A., and Mosher, R.S. A versatile walking truck. In *Proceedings of the Transportation Engineering Conference*. Institution of Civil Engineers, London, 1968.
13. Lucas, E. Huitieme recreation—La machine a marcher. *Recreat. Math. 4* (1894), 198–204.
14. Matsuoka, K. A mechanical model of repetitive hopping movements. *Biomechanisms 5* (1980), 251–258.
15. McGhee, R.B. Some finite state aspects of legged locomotion. *Math. Biosci. 2* (1968), 67–84.
16. McGhee, R.B. Vehicular legged locomotion. In *Advances in Automation and Robotics*, G.N. Saridis, Ed. JAI Press, 1983.
17. McGhee, R.B., and Frank, A.A. On the stability properties of quadruped creeping gaits. *Math. Biosci.* 3 (1968), 331–351.
18. McGhee, R.B., and Jain, A.K. Some properties of regularly realizable gait matrices. *Math. Biosci. 13* (1972), 179–193.
19. McGhee, R.B., and Kuhner, M.B. On the dynamic stability of legged locomotion systems. In *Advances in External Control of Human Extremities*, M.M. Gavrilovic and A.B. Wilson, Jr., Eds. Jugoslav Committee for Electronics and Automation, Belgrade, Yugoslavia, 1969, pp. 431–442.
20. Miura, H., and Shimoyama, I. Dynamic walk of a biped. *Int. J. Robotics Res.* 3 (1984), 60–74.
21. Morrison, R.A. Iron mule train. In *Proceedings of Off-Road Mobility Research Symposium*. International Society for Terrain Vehicle Systems, Washington, D.C., 1968, pp. 381–400.
22. Murphy, K.N., and Raibert, M.H. Trotting and bounding in a planar two-legged model. In *5th Symposium on Theory and Practice of Robots and Manipulators*, A. Morecki, G. Bianchi, and K. Kedzior, Eds. MIT Press, Cambridge, Mass., 1985, pp. 411–420.
23. Murthy, S.S., and Raibert, M.H. 3D balance in legged locomotion: Modeling and simulation for the one-legged case. In *Inter-Disciplinary Workshop on Motion: Representation and Perception*. (Toronto, Canada, Apr.). ACM, New York, 1983, pp. 193–200.
24. Muybridge, E. *The Human Figure in Motion*. Dover Publications, New York, 1955. First ed., Chapman and Hall, London, 1901.
25. Muybridge, E. *Animals in Motion*. Dover Publications, New York, 1957. First ed., Chapman and Hall, London, 1899.
26. Raibert, M.H. *Legged Robots that Balance*. MIT Press, Cambridge, Mass., 1986.
27. Raibert, M.H. Symmetry in running. *Science 231* (1986), 1292–1294.
28. Raibert, M.H., and Brown, H.B., Jr. Experiments in balance with a 2D one-legged hopping machine. *ASME J. Dyn. Syst. Meas. Control 106* (1984), 75–81.
29. Raibert, M.H., Brown, H.B., Jr., and Chepponis, M. Experiments in balance with a 3D one-legged hopping machine. *Int. J. Robotics Res.* 3 (1984), 75–92.
30. Raibert, M.H., Chepponis, M., and Brown, H.B., Jr. Running on four legs as though they were one. *IEEE J. Robotics Autom.* (1986), 2.
31. Russell, M., Jr. Odex I: The first functionoid. *Robotics Age* 5 (1983), 12–18.
32. Schaefer, J.F., and Cannon, R.H., Jr. *On the Control of Unstable Mechanical Systems*. International Federation of Automatic Control, London, 6c.1–6c.13.
33. Seifert, H.S. The lunar pogo stick. *J. Spacecr. Rockets* 4 (1967), 941–943.
34. Shigley, R. The mechanics of walking vehicles. Rep. 7, Land Locomotion Laboratory, Detroit, Mich., 1957.
35. Sitek, G. Big Muskie. *Heavy Duty Equip. Maint. 4* (1976), 16–23.
36. Snell, E. Reciprocating load carrier. Patent 2,430,537, 1947.
37. Sutherland, I.E., and Ullner, M.K. Footprints in the asphalt. *Int. J. Robotics Res.* 3 (1984), 29–36.
38. Urschel, W.E. Walking tractor. Patent 2,491,064, Valparaiso, Ind., 1949.
39. Vukobratovic, M., and Stepaneko, Y. On the stability of anthropomorphic systems. *Math. Biosci. 14* (1972), 1–38.
40. Waldron, K.J., Vohnout, V.J., Pery, A., and McGhee, R.B. Configuration design of the adaptive suspension vehicle. *Int. J. Robotics Res.* 3 (1984), 37–48.
41. Wallace, H.W. Jumping tank vehicle. Patent 2,371,368, Freeport, N.Y., 1942.

**CR Categories and Subject Descriptors:** I.2.9 [**Artificial Intelligence**]: Robotics—*propelling mechanisms*

**General Terms:** Design, Experimentation

**Additional Key Words and Phrases:** balance, legged machines, locomotion, running

Author's Present Address: Marc H. Raibert, Dept. of Computer Science and The Robotics Institute, Carnegie-Mellon University, Pittsburgh, PA 15213.