Using reputation and misinformation to influence users in selfish routing

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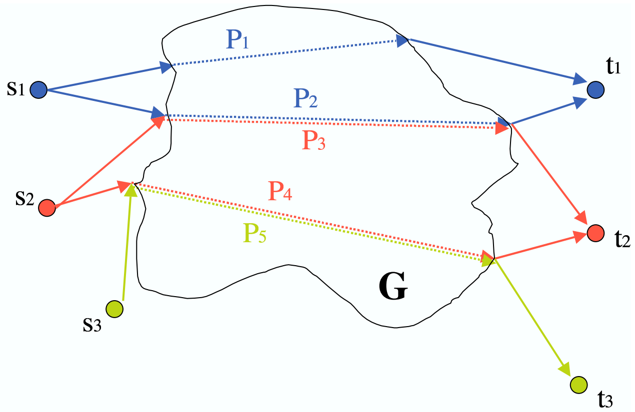
The model



Terminology

- r_i: flow rate for user (commodity) i.
- f_P: flow through path P
- $l_P(f)$: latency of P due to flow f.
  Additive model : $l_P(f) = \sum_{e \in P} l_e(f_e)$.
- $c_e(f_e)$: cost due to edge e.
  Total social cost : $\sum_{e \in E} c_e(f_e) = \sum_{e \in E} f_e l_e(f_e)$.

Optimization (coordinated) version

$$\min_f \sum_{e \in E} c_e(f_e) \quad \text{subject to:} \qquad (\text{MIN})$$

$$\sum_{P \in \mathcal{P}_i} f_P = r_i \qquad \forall i \in \{1, \ldots, k\}$$

$$f_e = \sum_{P \ni e} f_P \qquad \forall e \in E$$

$$f_P \geq 0 \qquad \forall P \in \mathcal{P}$$

Selfish (uncoordinated) version

- T_P: general disutility for path P as seen by its user (this may *not* be the same as l_P).
- Every user tries to minimize his overall general disutility $\Rightarrow$ traffic equilibrium.
- **Wardrop's principle for traffic equilibria:**
  At equilibrium, for each origin-destination pair the general disutilities on all the paths actually used are equal, and less than the general disutilities on all unused paths.

Coordination ratio [KP99]

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Known results for the case $T_P := l_P$ [RT02]

- If f is traffic equilibrium for (G, r, l) and $\hat{f}$ is optimum for $(G, 2r, l)$, then $Cost(f) \leq Cost(\hat{f})$.
- For linear latencies l_e, $\rho(G, r, l) \leq 4/3$.

Selfish vs. Optimal total latency

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- Question: Can we induce the selfish users to produce a flow that achieves *minimum* total latency (i.e., make $\rho = 1$)?
- Answer: Use *taxation* on the network edges:

$$T_P(f) := l_P(f) + a(i) \cdot \sum_{e \in P} \tau_e.$$

$a(i)$ is a *tax sensitivity* factor for user class i,
τ_e is the *per flow unit tax* on edge e.

Introducing taxation

Theorem 1 [HY03, FJM04, KK04]
$\exists \tau$ s.t. $\exists$ equilibrium f^* and $f^*_e = \hat{f}_e$, $\forall e \in E$. Moreover, τ can be calculated in poly-time if $\hat{f}$ is *given*.

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2. Artificial delay (but the latencies are now T_P...*bad!*)
3. A lie!

A scenario of incomplete information

Assume **linear** edge latencies and **single commodity** networks, and **marginal costs** as optimal taxes, i.e.,

$$T_e := a_e f_e + b_e + a_e f_e^{opt}.$$

A scenario of incomplete information

NETWORK

USERS

a_e , b_e , D

COMPLETE INFO

INCOMPLETE INFO

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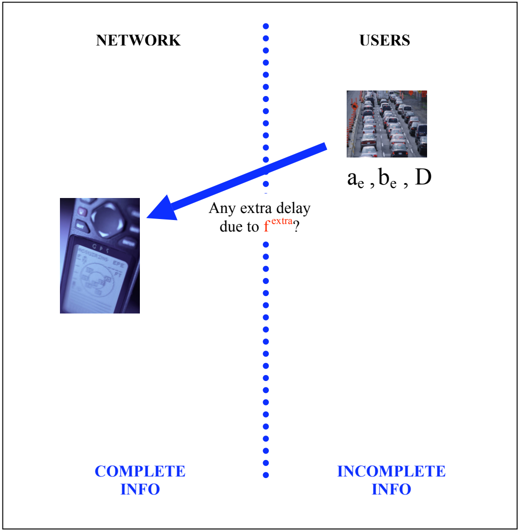

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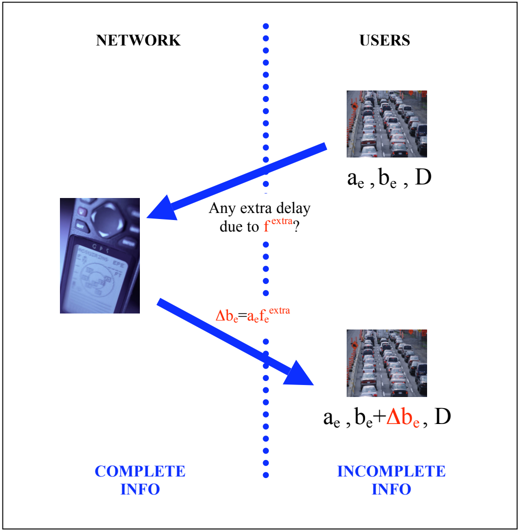
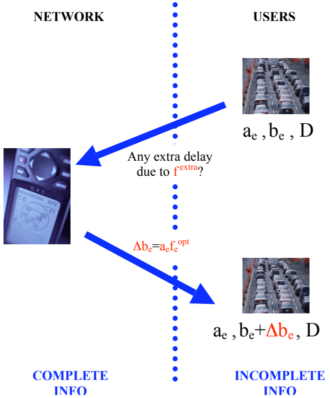


Lying repeatedly



$$\rho = 1$$

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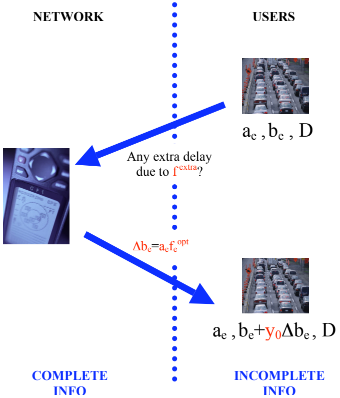

$$y_0 = 1$$
$$\rho_0 = 1$$

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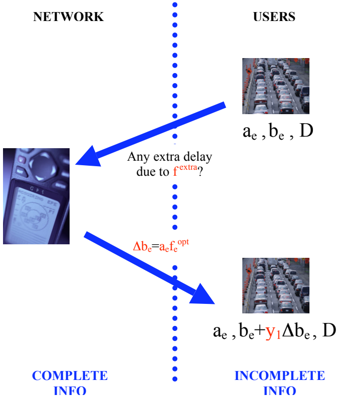

$$y_0 = 1 \quad > y_1$$
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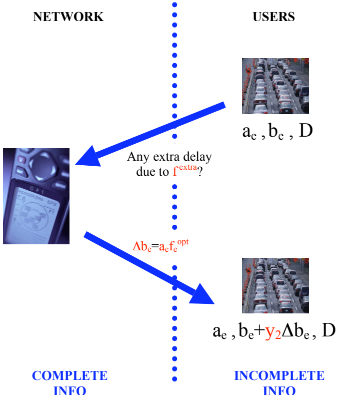

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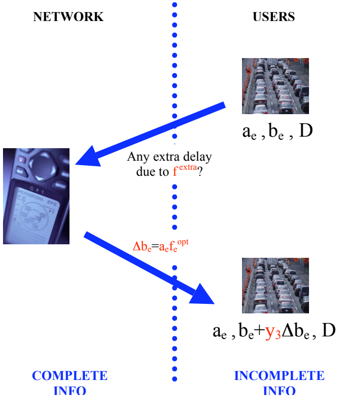

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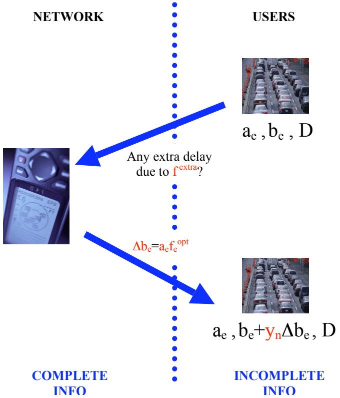
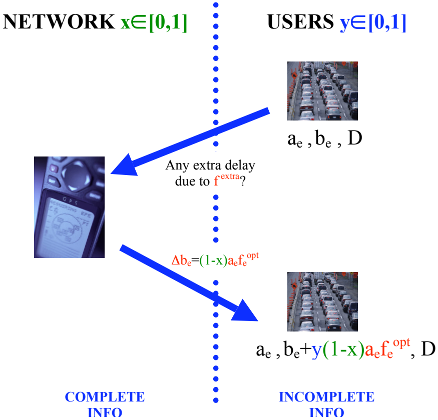


$$
\begin{array}{llllll}
y_0 = 1 & > y_1 & > y_2 & > y_3 & > \dots & > y_n \approx 0 \\
\rho_0 = 1 & < \rho_1 & < \rho_2 & < \rho_3 & < \dots & < \rho_n \approx \frac{4}{3}
\end{array}
$$

Lying repeatedly...and less stupidly



$$\rho \leq \frac{4}{3+(1-x)y}$$

The stage game

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1. Pick $x \in [0, 1]$.
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Assumption:

- $\Gamma(x,0) > L_0, \forall x \in [0,1)$
- $\Gamma(1,y) = L_0, \forall y \in [0,1]$

The repeated game

History $h^t = \{(x_0, y_0), (x_1, y_1), \ldots, (x_{t-1}, y_{t-1})\}$.

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TYPES FOR PLAYER 1

- **committed type ω_c:** Player 1 always plays $c \in (0, 1]$.
- **rational type ω_0:** Player 1 is not restricted.

Folk theorem for Nash Equilibria

Assumption

Function $g_2(x, y) = -\Gamma(x, y)$ *is continuous.*

Theorem ([FL89])

If $0 < \mu^* < 1$, *then for all* $\varepsilon > 0$ *there exists a* $\underline{\delta} < 1$ *such that for all* $\delta \in (\underline{\delta}, 1)$

$$\underline{V}_1^{NE}(\delta, \mu^*) \geq (1 - \varepsilon) g_1^* - \frac{4\varepsilon}{3}.$$

where

- μ^* : Player 2's initial belief on Player 1's type.
- g_1^* : Player 1's *Stackelberg payoff*.

Folk theorem for Perfect Bayesian Equilibria

Assumption

For any (mixed) action x (ν) by Player 1, Player 2 has a unique pure best response $y^*(x)$ ($y^*(\nu)$), and $y^*(\nu)$ increases if ν increases in the first-order stochastic dominance sense.

Theorem ([LS09])

For any $\varepsilon > 0$, $\mu^* \in (0,1)$, and $\delta > \frac{g_1(0,y^*(c)) - g_1^*}{g_1(0,y^*(c)) - 4/3}$, there exists integer $K(\varepsilon, \mu^*)$ such that if record keeping length $K > K(\varepsilon, \mu^*)$, Player 1's payoff is lower bounded at any time by

$$\delta^K g_1^* + (1 - \delta^K) - \left[\frac{4(1-\delta^K)}{3} + \varepsilon\right]$$

which converges to $g_1^* - \varepsilon$ as δ goes to 1.

Note: Assumption only for technical reasons.

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- [LS09] is based on the bounded rationality of Player 2.
- [LS09] is powerful enough to also provide the *strategy* of Player 1: play $x = c$ for $K - 1$ rounds, and then play $x = 0$.

Open problems

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- Models of *incomplete information*.