

Algorithms

Assignment: Divide and Conquer

Name:

Id:

Grade

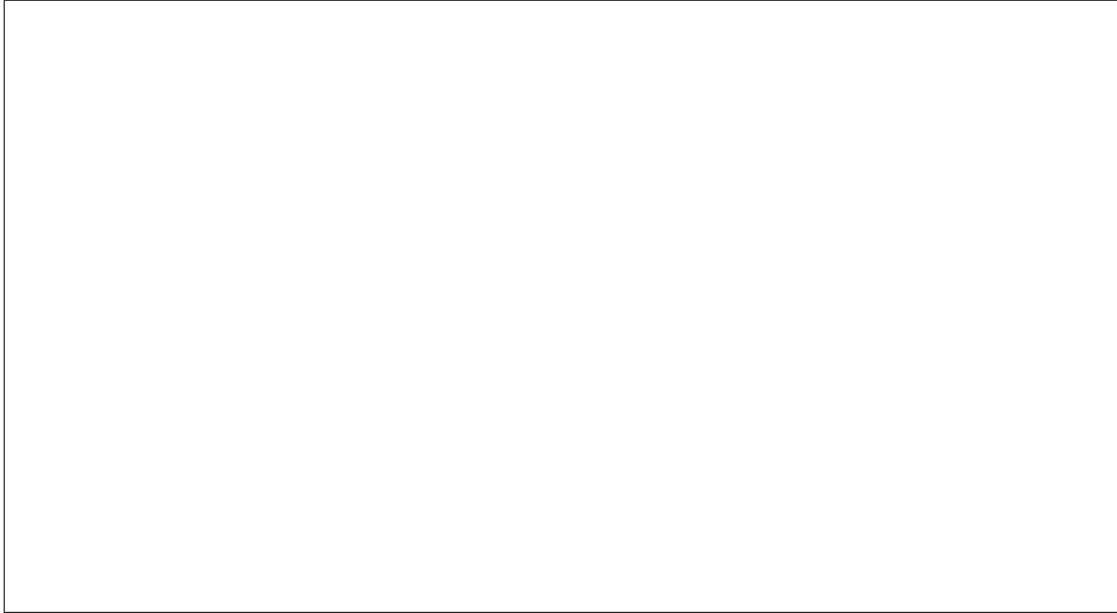
Good Luck!

Matrix multiplication:

- With the direct method, it is possible to multiply 2 matrices of size $n \times n$ with exactly n^3 scalar multiplications and $n^2(n - 1)$ scalar additions.
- With the Strassen method, it is possible to multiply 2 matrices of size 2×2 with exactly 7 scalar multiplications and 18 scalar additions.
- The complexity for multiplying 2 matrices of size $n \times n$ using the Strassen algorithm is $O(n^{\log_2 7}) \approx O(n^{2.81})$.

1. Use the recursive Strassen method to compute the exact number of scalar multiplications and scalar additions to multiply 2 matrices of size 4×4 .

Explain how you computed the two numbers.

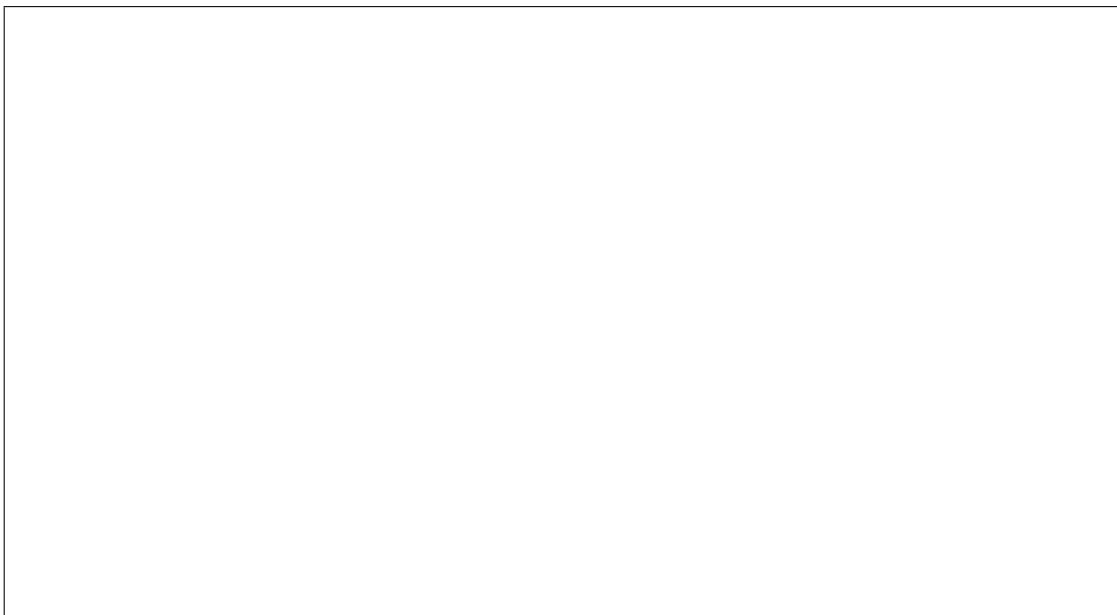


2. Let $M(n)$ be the number of multiplications used by the Strassen algorithm for $n = 2^k$ ($k \geq 1$).

What is the recursive formula for $M(n)$?

What is the **exact** solution for $M(n)$?

Note that in this problem you ignore the number of additions.

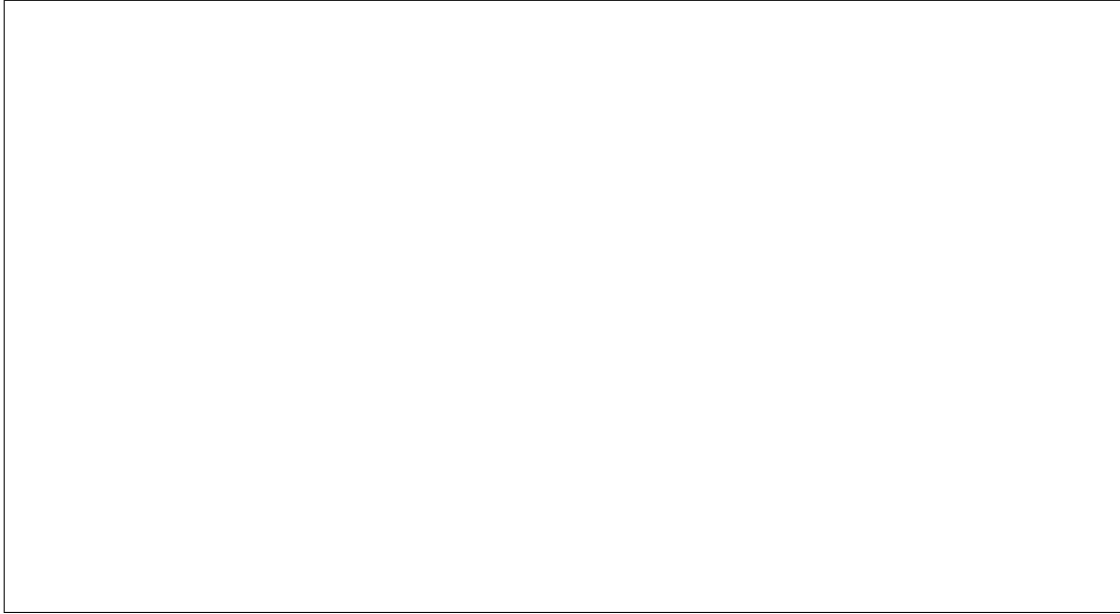


3. Let $A(n)$ be the number of additions used by the Strassen algorithm for $n = 2^k$ ($k \geq 1$).

What is the recursive formula for $A(n)$?

What is the **exact** solution to $A(n)$?

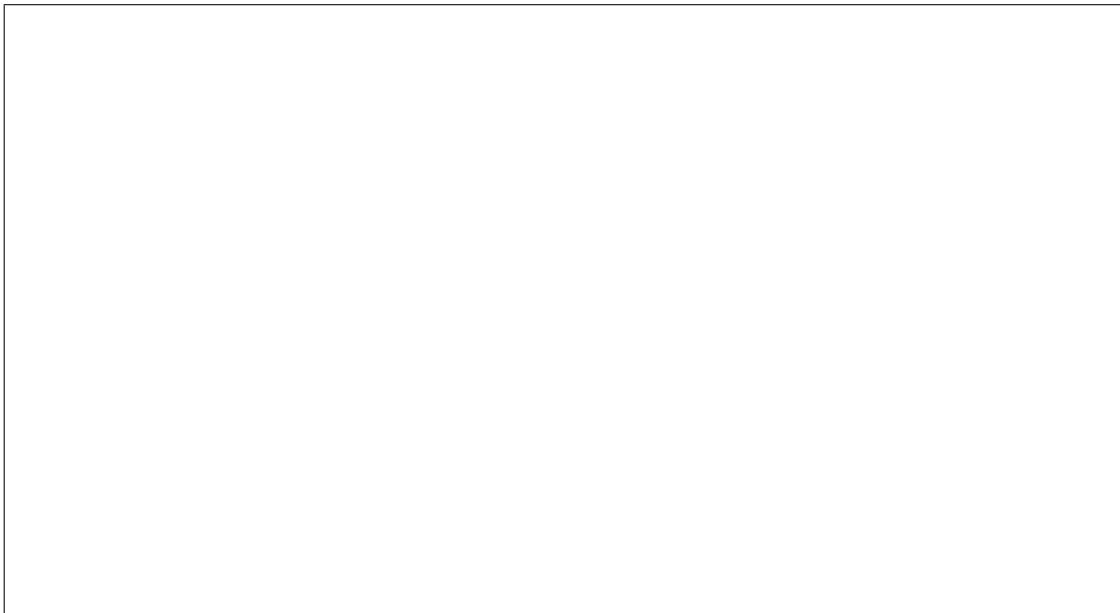
Note that in this problem you ignore the number of multiplications.



4. Assume that n is a power of 3. Assume that there is a way to multiply 2 matrices of size 3×3 with 25 scalar multiplications and 50 scalar additions.

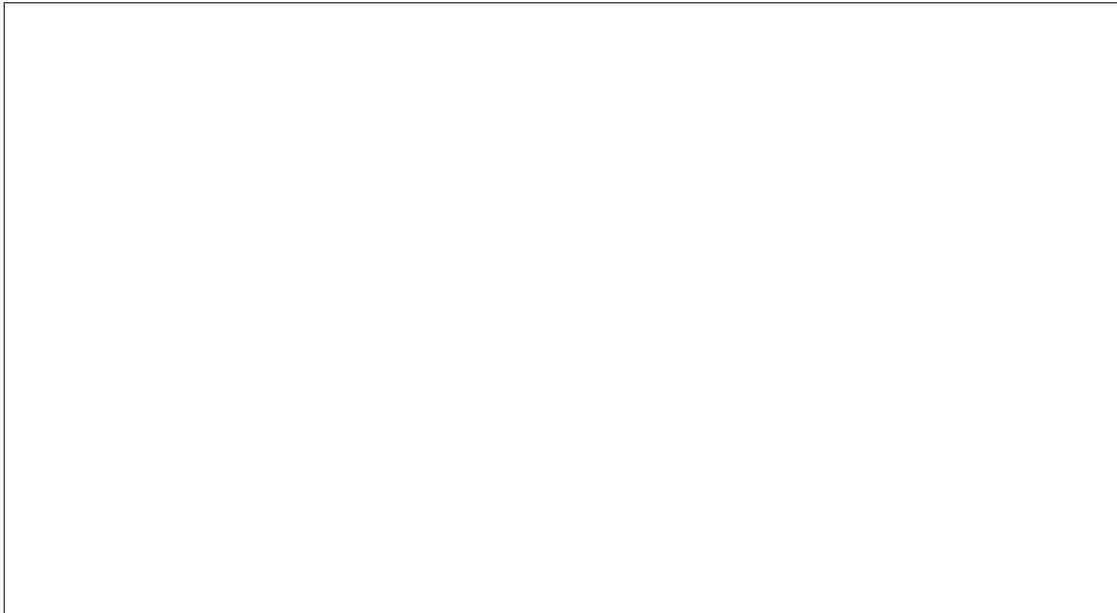
What is the complexity of a recursive algorithm that is based on this method?

Write the recursive formula and use the master theorem to solve it.



5. What should be the number of scalar multiplications in multiplying 2 matrices of size 3×3 in order to get a recursive way to multiply 2 matrices of size $n \times n$ with a better complexity than the original Strassen algorithm?

Explain your answer.



6. Assume that n is a power of 70. There exists a way to multiply 2 matrices of size 70×70 with 143640 scalar multiplications and α scalar additions for some constant α .

What is the complexity of a recursive algorithm that is based on this method?

Write the recursive formula and use the master theorem to solve it.

