Algorithms

Assignment: Graphs

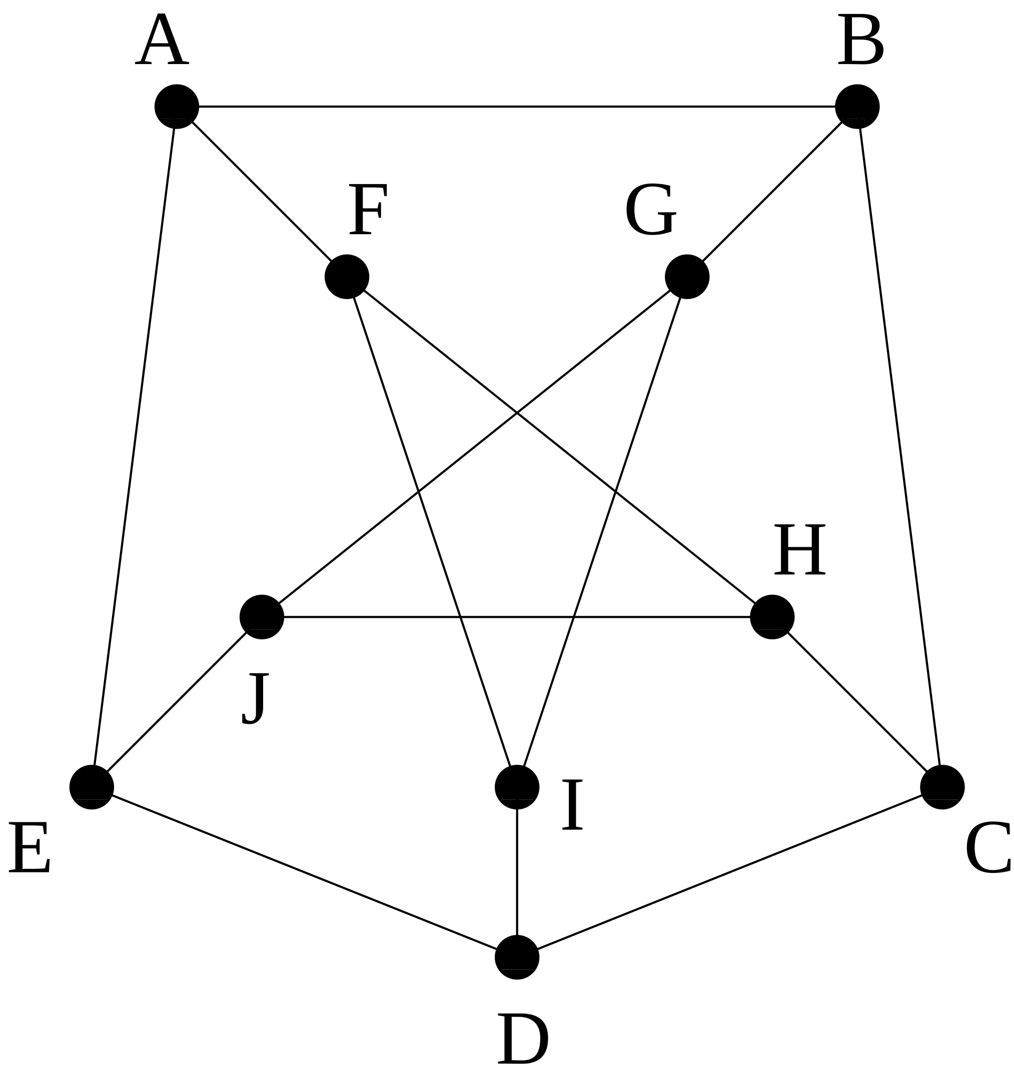
Name: ....

Id: ....

Grade

Good Luck!

1. Consider the following Petersen graph:



(a) Find 2 additional "nice" drawings of this graph.

(b) Fill the adjacency matrix of the Petersen graph.

	A	B	C	D	E	F	G	H	I	J
A										
B										
C										
D										
E										
F										
G										
H										
I										
J										

(c) Fill the adjacency lists of the Petersen graph.

$A \rightarrow (\qquad)$

$B \rightarrow (\qquad)$

$C \rightarrow (\qquad)$

$D \rightarrow (\qquad)$

$E \rightarrow (\qquad)$

$F \rightarrow (\qquad)$

$G \rightarrow (\qquad)$

$H \rightarrow (\qquad)$

$I \rightarrow (\qquad)$

$J \rightarrow (\qquad)$

2. Special sets in a graph.

   (a) An **independent set** is a set of vertices that have no edges among them. What is the size of the **largest** possible independent set in the Petersen graph? Give an example of such a set.

   (b) Prove that a larger set does not exist.

   (c) A **clique** is a set of vertices that contains all the possible edges among them. What is the size of the **largest** possible clique in the Petersen graph? Give an example of such a clique.

   (d) Prove that a larger set does not exist.

(e) A **vertex cover** is a set of vertices where each edge has at least one of its vertices in the set. What is the size of the **smallest** possible vertex cover in the Petersen graph? Give an example of such a set.

(f) Prove that a smaller set does not exist.

(g) A **dominating set** is a set of vertices where all other vertices have at least one neighbor in this set. What is the size of the **smallest** possible dominating set in the Petersen graph? Give an example of such a set.

(h) Prove that a smaller set does not exist.

3. Paths and cycles in a graph.

A **path** $\mathcal{P} = \langle v_0, v_1, \ldots, v_k \rangle$ of length k is an ordered list of vertices such that the edge (v_i, v_{i+1}) exists for $0 \le i \le k-1$ and all the edges are different.

A **cycle** $\mathcal{C} = \langle v_0, v_1, \ldots, v_{k-1}, v_0 \rangle$ of length k is a path that starts and ends with the same vertex.

In a **simple path** all the vertices are different.
In a **simple cycle** all the vertices except $v_0 = v_k$ are different.

An **Euler path** is a path that contains all the edges in the graph.
An **Euler cycle** is a cycle that contains all the edges.

A **Hamiltonian path** is a simple path that contains all the vertices in the graph.
A **Hamiltonian cycle** is a simple cycle that contains all the vertices in the graph.

The Petersen graph does not have neither an Euler path nor an Euler cycle.
It has a Hamiltonian path but not a Hamiltonian cycle.

(a) Find one of the longest paths (does not have to be simple) in the Petersen graph.

(b) Find one of the longest cycles (does not have to be simple) in the Petersen graph.

(c) Find one of the Hamiltonian paths in the Petersen graph.

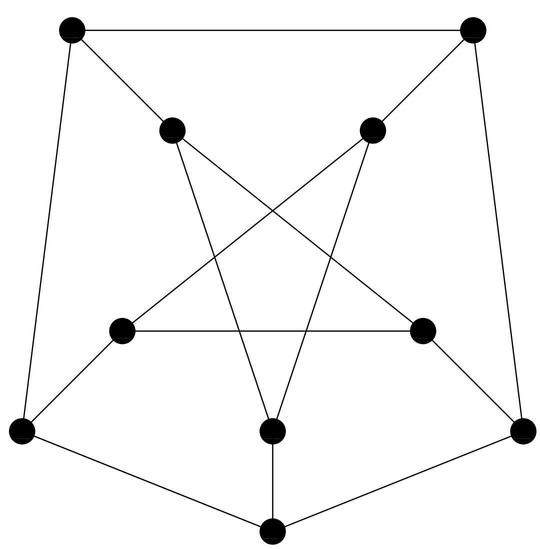
(d) Find one of the longest simple cycles in the Petersen graph.

4. A coloring of a graph is an assignment of “colors” to the vertices such that the colors assigned to the two vertices of any edge are different.

(a) How many colors are needed to color a complete graph with n vertices? Explain your answer.

(b) How many colors are needed to color a bipartite graph? Explain your answer.

(c) Color the Petersen graph with as few colors as possible.



5. Trees and bipartite graphs.

(a) Prove or disprove: every tree is a bipartite graph.

(b) Prove or disprove: every bipartite graph is a tree.

6. Illustrate the 6 possible molecules of the type C_4H_y in which the connection between two C can have at most one edge.

7. Prove that for every $n \geq 5$ there exists a graph with n vertices, all of which have degree 4.

8. Prove that no graph has all the degrees different; that is, prove that in a degree sequence there is at least one repeated number.

9. Let $G = (V, E)$ be a simple undirected graph. $\tilde{G} = (\tilde{V}, \tilde{E})$ is the complement graph of G if $V = \tilde{V}$ and $(x, y) \in E$ if and only if $(x, y) \notin \tilde{E}$.

   Prove that either G is connected or $\tilde{G}$ is connected.