

Algorithms

Assignment: Greedy Algorithms

Name:

Id:

Grade

Good Luck!

Scheduling Jobs on Machines:

- There are n jobs: J_1, \dots, J_n .
- There are m machines: M_1, \dots, M_m .
- Job J_i is represented by the time interval: $[s_i, f_i)$; $f_i > s_i \geq 0$ for all $1 \leq i \leq n$.
- Job J_i is associated with a profit value $p_i > 0$.
- Mutual Exclusion: at any point of time $t \geq 0$, a machine can serve at most one job.
- Objective: schedule the jobs on the machines to optimize some objective function.

A greedy scheme:

1. Order the jobs according to some greedy criterion.
2. Scan the jobs according to this order.
3. When considering a job, make a decision whether to accept it or to reject it.
4. In case of acceptance, select a machine for this job obeying the mutual exclusion requirement.
5. Decisions are irreversible: a rejected job cannot be scheduled later and an accepted job cannot be preempted.

Remark: If possible, illustrate your answers.

1. The activity selection problem:

- There is only one machine.
- The profit of each job is 1.
- The goal is to maximize the profit. This is equivalent to finding the largest set of jobs that can be scheduled on one machine.
- Ordering by a non-decreasing order of the finish times produces an optimal solution.

(a) Show an input for which ordering by a non-decreasing order of the starting times does not produce an optimal solution.

If you can, show that for an infinite values of n , there are inputs for which this greedy algorithm produces a solution whose profit is almost $1/n$ of the optimal profit.

(b) Show an input for which ordering by length (from the shortest to the longest) does not produce an optimal solution.

If you can, show that for infinite values of n , there are inputs for which this greedy algorithm produces a solution whose profit is almost half of the optimal profit.

- (c) The degree of a job J_i is the number of jobs whose time intervals intersect with the time interval of J_i .
Show an input for which ordering by degree (from the smallest to the largest) does not produce an optimal solution.

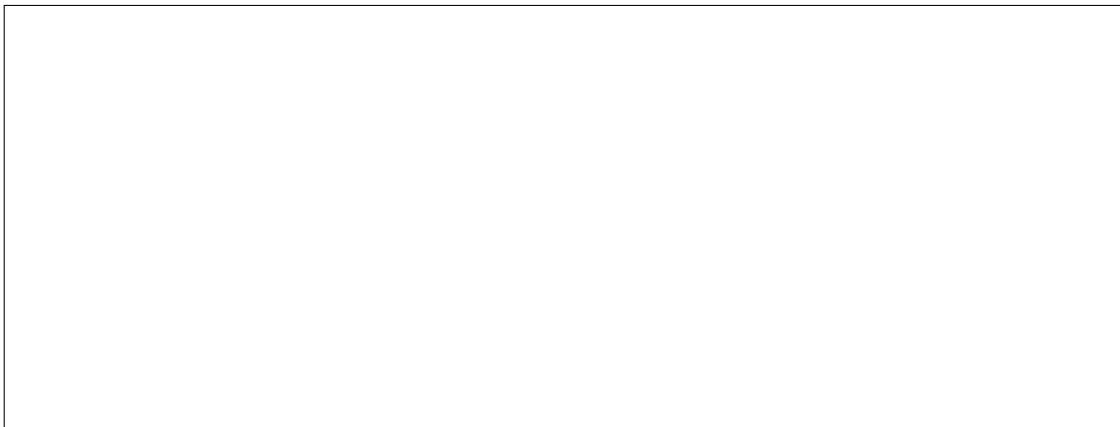


2. The profit of a job is its length:

- There is only one machine.
- The profit of a job is its length: $p_i = f_i - s_i$.
- The goal is to maximize the profit. This is equivalent to finding a set of jobs whose intervals “covers” the time interval the most.

Show an input for which ordering by length (from longest to shortest) is not optimal.

If you can, show that for infinite values of n , there are inputs for which this greedy algorithm produces a solution whose profit is almost one third of the optimal profit.



3. Arbitrary profits:

- There is only one machine.
- The profit of a job is arbitrary (shorter jobs may be more profitable than longer jobs).
- The goal is to maximize the profit. This is equivalent to finding a set of jobs whose total profit is the largest.

(a) Show an input for which ordering by profit (from the most profitable to the least profitable) is not optimal.

If you can, show that for infinite values of n , there are inputs for which this greedy algorithm produces a solution whose profit is almost $1/n$ of the optimal profit.

(b) Show an input for which ordering by the ratio profit over length (from the largest ratio to the smallest ratio) is not optimal.

If you can, show that for infinite values of n , there are inputs for which this greedy algorithm produces a solution whose profit is almost one third of the optimal profit.

4. Must schedule all the jobs:

- The goal is to schedule all the jobs with minimum number of machines (the profit is irrelevant).
- In the greedy scheme, whenever a job is considered, it is scheduled on the first possible machine (first-fit-greedy). When none of the existing machines can serve this job, a new machine is added to serve this job.

(a) Show an input for which ordering by a non-decreasing order of the finish times does not produce an optimal solution.

(b) Prove that ordering by a non-decreasing order of the starting times produces an optimal solution.