Algorithms

Assignment: Order Statistics

Name: .....

Id: .....

Grade

Good Luck!

1. Let $A[1], A[2], \ldots, A[n]$ be an array containing n positive integers.

   Describe an *efficient* algorithm to find the *maximum* difference between any two integers in the array. In other words, compute $M = \max_{1 \le i,j \le n} \{A[i] - A[j]\}$.

   What is the complexity of your algorithm? Explain. If you know the exact complexity write it. Otherwise, you may use the "O, Ω, Θ" notation.

2. Let $A = A[1], \ldots, A[n]$ be an array of $n \geq 4$ distinct keys.

   Describe an *efficient* algorithm to find the **three** smallest keys in A. Use words.

   What is the *worst case* number of comparisons performed by your algorithm. Try to find an **exact** number. Ignore floors and ceilings. Explain your answer.

3. Design an efficient algorithm to find the median of 5 distinct keys.

4. Let A be an array containing n very large positive integers not necessarily distinct. A *majority* is a number that appears at least $\lceil (n+1)/2 \rceil$ times in the array (note that there can be at most one majority). Describe an $O(n)$-time algorithm that finds a majority in A **if exists**. Explain why your algorithm has time complexity $O(n)$.