Algorithms

Assignment: Binary Search

Name: ..........................................................

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Grade

Good Luck!
1. Assume $n \geq 1$ is a power of 2. Let $A = [A_1 \leq A_2 \leq \cdots \leq A_n]$ be a sorted array of $n$ integers. Let $x$ be an integer.

Design an efficient algorithm that finds the number of times $x$ appears in the array. What is the worst-case number of comparisons made by your algorithm?

Justify the correctness and complexity of your algorithm.
2. Assume \( n \geq 1 \) is a power of 2. Let \( A = [A_1 < A_2 < \cdots < A_n] \) be a sorted array of \( n \) distinct positive integers. Let \( x \leq y \) be two positive integers.

Describe an efficient algorithm that determines if one of the numbers \( x, x+1, \ldots, y \) appears in the array. What is the worst-case number of comparisons made by your algorithm? Justify the correctness and complexity of your algorithm.
3. Assume $n \geq 1$ is a power of 2. Let $A = [A_1 < A_2 < \cdots < A_n]$ be a sorted array of $n$ distinct positive integers. Let $x \leq y$ be two positive integers.

Describe an efficient algorithm that determines if all the numbers $x, x + 1, \ldots, y$ appear in the array. What is the worst-case number of comparisons made by your algorithm?

Justify the correctness and complexity of your algorithm.
4. Let \( A = [A_1 < A_2 < \cdots < A_n] \) be a sorted array of \( n \) distinct integers (could be positive and/or negative).

Describe an efficient algorithm that finds, if exists, an index \( 1 \leq i \leq n \) such that \( A_i = i \).

What is the worst-case number of comparisons made by your algorithm?

Justify the correctness and complexity of your algorithm.
5. Let $A = [A_1 < A_2 < \cdots < A_n]$ be an array of $n$ distinct integers sorted in an **ascending order** and let $B = [B_1 > B_2 > \cdots > B_n]$ be an array of $n$ distinct integers sorted in a **descending** order.

Describe an efficient algorithm that finds, if exists, an index $1 \leq i \leq n$ such that $A_i = B_i$.

What is the worst-case number of comparisons made by your algorithm?

Justify the correctness and complexity of your algorithm.