Algorithms

Assignment: Matrix Multiplication

Complexity:

- With the direct method, it is possible to multiply two matrices of size $n \times n$ with exactly $n^3$ scalar multiplications and $n^2(n - 1)$ scalar additions.
- With the Strassen method, it is possible to multiply two matrices of size $2 \times 2$ with exactly 7 scalar multiplications and 18 scalar additions.
- The complexity for multiplying two matrices of size $n \times n$ using the Strassen algorithm is

$$\Theta\left(n^{\log_2 7}\right) = \Theta\left(n^{2.807354922...}\right) = O\left(n^{2.81}\right)$$

Problems:

1. Use the recursive Strassen method to compute the exact number of scalar multiplications and scalar additions (subtractions are considered as additions) to multiply two matrices of size $4 \times 4$, two matrices of size $8 \times 8$, and two matrices of size $16 \times 16$. Explain how you computed these numbers.

2. Let $M(n)$ be the number of multiplications used by the Strassen algorithm for $n = 2^k$ ($k \geq 1$). What is the recursive formula for $M(n)$? What is the exact solution for $M(n)$? Note that in this problem the number of additions is ignored.

3. Let $A(n)$ be the number of additions used by the Strassen algorithm for $n = 2^k$ ($k \geq 1$). What is the recursive formula for $A(n)$? What is the exact solution to $A(n)$? Note that in this problem the number of multiplications is ignored.

4. Assume that $n$ is a power of 3. Assume that there is a way to multiply two matrices of size $3 \times 3$ with 25 scalar multiplications and 50 scalar additions. What is the complexity of a recursive algorithm that is based on this method? Write the recursive formula and use the master theorem to solve it.

5. What should be the number of scalar multiplications in multiplying two matrices of size $3 \times 3$ in order to get a recursive way to multiply two matrices of size $n \times n$ with a better complexity than the original Strassen algorithm? Justify your answer.

6. Assume that $n$ is a power of 70. There exists a way to multiply two matrices of size $70 \times 70$ with 143640 scalar multiplications and $\alpha$ scalar additions for some constant $\alpha$. What is the complexity of a recursive algorithm that is based on this method? Write the recursive formula and use the master theorem to solve it.