CISC 7200X – Analysis of Algorithms

Realizing Degree Sequences Programming Project – Testing

Testing your program:

- Six tasks are described in the next page. Each task contains several (already sorted) input sequences to test your program.

- For your final report, do as many tasks as you can (not necessarily in their order).

- For the first four tasks, you are also asked, if you can, to construct the output graph without running your program.

- Do the same tasks for each variant of the HH algorithm that you implement.

Remarks:

- If a sequence is not realizable, your program should announce this.

- If the sequence is realizable, show the graph produced by your program if it has an interface that illustrates the output graph. Otherwise, show the adjacency matrix or the adjacency lists that your program outputs and illustrate the graph yourself.

- Call the vertices $A, B, C, \ldots$ (the longest sequence has less than 27 vertices). Then the sequence $(d_1 \geq d_2 \geq \cdots \geq d_n)$ becomes

$$
(d(A) \geq d(B) \geq d(C) \geq \cdots)
$$

- In the illustrations, the names of the vertices should appear inside the circles that represent these vertices (the degrees may appear near the vertices). The original degree sequence should appear below the illustration.
Tasks:

1. Run your program on 2-regular graphs. For \( n \geq 3 \), the input sequence is \((2, 2, \ldots, 2)\) in which 2 appears \( n \) times.
   (a) What is the output of the algorithm for \( n = 6, 7, 8, 9, 10, 11, 12\)?
   (b) Can you find the answer for a general \( n \) without running the algorithm?

2. Run your program on 3-regular graphs. For \( n \geq 4 \), the input sequence is \((3, 3, \ldots, 3)\) in which 3 appears \( n \) times for an even \( n \).
   (a) What is the output of the algorithm for \( n = 4, 6, 8, 10, 12, 14, 16\)?
   (b) Can you find the answer for a general \( n \) without running the algorithm?

3. For an even \( n = 2k \), run your program on the following sequence:
   \((k, k, k - 1, k - 1, \ldots, 2, 2, 1, 1)\)
   in which each of the numbers 1, 2, \ldots, \( k \) appears exactly twice.
   (a) What is the output of the algorithm for \( n = 2, 4, 6, 8, 10, 12\)?
   (b) Can you find the answer for a general \( n \) without running the algorithm?

4. For \( n \geq 2 \) and \( 1 \leq i \leq n - 1 \), let
   \[ S(n, i) = (n - 1, n - 2, \ldots, i + 1, i, i, i - 1, \ldots, 1) \]
   That is, \( S(n, i) \) is a sequence of length \( n \) that contains all the numbers 1, 2, \ldots, \( n - 1 \) while the number \( i \) is the only one that appears twice in \( S(n, i) \). Run your program on \( S(n, i) \) sequences
   (a) What is the output of the algorithm for \( n = 6 \) and \( i = 1, 2, 3, 4, 5 \)?
   (b) What is the output of the algorithm for \( n = 7 \) and \( i = 1, 2, 3, 4, 5, 6 \)?
   (c) What is the output of the algorithm for \( n = 8 \) and \( i = 1, 2, 3, 4, 5, 6, 7 \)?
   (d) What is the output of the algorithm for \( n = 9 \) and \( i = 1, 2, 3, 4, 5, 6, 7, 8 \)?
   (e) Can you find the answer for general \( n \) and \( i \) without running the algorithm?

5. Run your program on the following sequences:
   (a) \((3, 3, 3, 3, 3, 3, 3, 3)\)
   (b) \((3, 3, 3, 3, 3, 3, 2, 2)\)
   (c) \((3, 3, 3, 3, 2, 2, 2)\)
   (d) \((3, 3, 2, 2, 2, 2, 2)\)
   (e) \((2, 2, 2, 2, 2, 2, 2)\)

6. Run your program on the following sequences:
   (a) \((2, 2, 1, 1)\)
   (b) \((3, 3, 3, 2, 2, 2, 1, 1, 1)\)
   (c) \((4, 4, 4, 4, 3, 3, 3, 3, 2, 2, 2, 2, 1, 1, 1, 1)\)