Algorithms

Quiz: Huffman Codes

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The following figure illustrates the 5 possible binary trees that represent Huffman codes for the letters \( \{a, b, c, d\} \) for which \( f(a) > f(b) > f(c) > f(d) \). The trees are denoted by \( T_1, T_2, T_3, T_4, T_5 \).

\[
\begin{align*}
&\begin{array}{cccc}
\text{letter} & a & b & c & d \\
C_1 & 0 & 11 & 101 & 100 \\
C_2 & 1 & 01 & 001 & 000 \\
C_3 & 1 & 00 & 010 & 011 \\
C_4 & 1 & 00 & 011 & 010 \\
C_5 & 11 & 10 & 01 & 00 \\
C_6 & 10 & 11 & 01 & 00 \\
C_7 & 0 & 10 & 111 & 110
\end{array}
\end{align*}
\]

Match 5 out of the 7 codes with the 5 trees in the following table. Recall, that a left turn is 0 and a right turn is 1.
The following figure illustrates the 5 possible binary trees that represent Huffman codes for the letters \( \{a, b, c, d\} \) for which \( f(a) > f(b) > f(c) > f(d) \). The trees are denoted by \( T_1, T_2, T_3, T_4, T_5 \).

b. (25) The following are 5 possible frequency vectors denoted by \( F_1, F_2, F_3, F_4, F_5 \):

<table>
<thead>
<tr>
<th>letter</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_1 )</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>( F_2 )</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>( F_3 )</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>( F_4 )</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>( F_5 )</td>
<td>10</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Using the Huffman-Code algorithm, match the frequency vectors with the 5 trees in the following table. Recall that the algorithm assigns the smaller frequency to the left child and the larger frequency to the right child.
The following is a balanced tree with $n = 2^k$ leaves denoted by $a_1, a_2, \ldots, a_n$. Note that all the codewords are of length $k$.

c. (25) Find a frequency vector in which each letter has a distinct frequency and for which the Huffman-Code algorithm generates this balanced tree. Specify the frequency of the letter $a_i$ for $1 \leq i \leq n$ (try to find a “nice” vector).
The following is one of the “tallest” possible binary tree with $n$ leaves denoted by $a_1, a_2, \ldots, a_n$.

\begin{center}
\begin{tikzpicture}[scale=0.8]
\node (a1) at (0,0) {a_1};
\node (a2) at (-1,-1) {a_2};
\node (a3) at (-2,-2) {a_3};
\node (a4) at (-3,-3) {a_4};
\node (a5) at (-4,-4) {a_5};
\node (an) at (-5,-5) {a_n};
\node (an-1) at (-4,-5) {a_{n-1}};
\node (an-2) at (-3,-5) {a_{n-2}};
\draw (a1) -- (a2);
\draw (a2) -- (a3);
\draw (a1) -- (a4);
\draw (a4) -- (a5);
\draw (a1) -- (a6);
\draw (a6) -- (a7);
\draw (a7) -- (a8);
\draw (a8) -- (a9);
\draw (a9) -- (a10);
\draw (a10) -- (a11);
\draw (a11) -- (a12);
\end{tikzpicture}
\end{center}

d. (25) Find a frequency vector in which each letter has a distinct frequency for which the Huffman-Code algorithm generates this non-balanced tree. Specify the frequency of the letter $a_i$ for $1 \leq i \leq n$ (try to find a “nice” vector).