Discrete Math

Quiz: Prerequisite

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1. Identify the five components of the famous formula $e^{\pi i} - 1 = 0$

(a) The additive identity: ______
(b) The square root of $-1$: ______
(c) The multiplicative identity: ______
(d) The base of the natural logarithmic: ______
(e) The ratio of a circle’s circumference to its diameter: ______

2. Order the following five numbers in an increasing order: $e, \sqrt{2}, \phi, 1, \pi$. By definition, $e$ is the base of the natural logarithm, $\phi$ is the golden ratio, and $\pi$ is the ratio of a circle’s circumference to its diameter.

   ____ < ____ < ____ < ____ < ____

3. Let $A$ be the set of all the prime numbers between 10 and 30. Let $B$ be the set of all integers between 10 and 30 that are of the form $3k + 2$ for some integer $k$. Find the following sets:

(a) $A =$ __________________________
(b) $B =$ __________________________
(c) $A \cup B =$ __________________________
(d) $A \cap B =$ __________________________
(e) $A \setminus B =$ __________________________
(f) $B \setminus A =$ __________________________

4. Find all possible True and False assignments for the 3 variables $x, y, z$ (out of the possible 8 assignments) that satisfy the following formula:

$$(x \lor y) \land (\overline{x} \lor z) \land (\overline{y} \lor \overline{z})$$

5. Expand the following expressions:

(a) $(x + y)^2 =$ __________________________
(b) $(x - y)^2 =$ __________________________
(c) $(x + y)^3 =$ __________________________
(d) $(x - y)^3 =$ __________________________
6. Factor the following expressions:
   (a) \( x^2 - y^2 = \) ________________
   (b) \( x^3 - y^3 = \) ________________
   (c) \( x^3 + y^3 = \) ________________

7. Simplify the following expressions:
   (a) \( \frac{(n+1)!}{(n-1)!} = \) ________________
   (b) \( \left\lfloor \frac{n}{k} \right\rfloor \times k + (n \mod k) = \) ________________
   (c) \( x^n \times x^m = \) ________________
   (d) \( x^n \times y^n = \) ________________
   (e) \( \frac{\log_a(x^n)}{\log_a(x)} = \) ________________

8. Answer the following questions:
   (a) If \( \log_a(y) = x \), then \( a^x = \) ________________
   (b) If \( \log_a(x) + \log_a(y) = \log_a(z) \), then \( z = \) ________________
   (c) If \( x < 0 \), then \( |x| + |−x| = \) ________________

9. Let \( x = 126 \) and \( y = 60 \).
   (a) What is the Greatest Common Divisor (GCD) of \( x \) and \( y \)? ________________
   (b) What is the Least Common Multiplier (LCM) of \( x \) and \( y \)? ________________

10. Order the following functions from the slowest to the fastest when \( n \) tends to infinity:
    \( n^2 \); \( \log(n) \); \( n \); \( 2^n \); \( n^n \)

11. Based on the following two linear equations, find the value of \( x \) and \( y \) as a function of \( a \).
    \[ x + y = a \]
    \[ 2x + 5y = 0 \]
12. What are the roots of the quadratic equations \( ax^2 + bx + c = 0 \)?

13. Let \( 0 \leq x \leq 1 \) be a real number and let \( f(x) = \lfloor x \rfloor + \lceil x \rceil \). For which values of \( x \), \( f(x) < 1 \), \( f(x) = 1 \), and \( f(x) > 1 \)?

14. Find the sum of the following sequences:
   
   (a) \( 1 + 2 + 3 + \cdots + n = \)

   (b) \( 1 + 2 + 4 + 8 + \cdots + 2^k = \)

15. When a fair coin is flipped, then both the probabilities of Head (H) and Tail (T) are \( \frac{1}{2} \). Three coins are flipped. What is the probability that
   
   (a) all are H or all are T: 
   
   (b) there is exactly one H: 

16. Let \( T \) be a right angle triangle with sides \( a \), \( b \), and \( c \) where \( c \) is the hypotenuse (the side opposite the right angle). Write \( c \) as a function of \( a \) and \( b \).

17. What is the sum of the degrees of all the inner angles of the following geometric shapes?

   (a) Triangle 

   (b) Square
18. Let $C$ be a circle whose radius is $r$.

(a) What is the circumference of $C$?  

(b) What is the area of $C$?  

19. Solve the following recursive formulas for $T(n)$. Express $T(n)$ as a function of $n$.

(a) $n \geq 1$ is an integer.

\[
T(1) = 1 \\
T(n) = T(n - 1) + 1
\]

(b) $n \geq 1$ is an integer.

\[
T(1) = 1 \\
T(n) = 2T(n - 1)
\]

20. What is the value of $c$ when each procedure terminates?

(a) $f(n)$ (* $n > 0$ is an integer number *)

\[
c = 0 \\
\text{for } i = 1 \text{ to } n \text{ do} \\
\quad \text{for } j = 1 \text{ to } n \text{ do} \\
\quad \quad c := c + 1
\]

(b) $f(n)$ (* $n > 0$ is an integer number *)

\[
c = 1 \\
\text{for } i = 1 \text{ to } n \text{ do} \\
\quad c := c \times 2
\]