Discrete Math

Sets Practice Problems

Name: .................................................................
Id: .................................................................

<table>
<thead>
<tr>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>
1. Let $A$ be the set of all the even positive integers less than 19. Let $B$ be the set of all the multiples of 3 positive integers less than 19. Define the following sets:

(a) $A =$ ____________________________
(b) $B =$ ____________________________
(c) $\neg A =$ ____________________________
(d) $\neg B =$ ____________________________
(e) $A \cup B =$ ____________________________
(f) $A \cap B =$ ____________________________
(g) $A \setminus B =$ ____________________________
(h) $B \setminus A =$ ____________________________
(i) $\neg (A \cup B) =$ ____________________________
(j) $\neg (A \cap B) =$ ____________________________

2. Write the two De Morgan’s laws for the sets $A$, $B$, $C$, and $D$ in three ways using the three negation notations for a set $X$: $X'$, $\overline{X}$, and $\neg X$. 
3. Show using the Venn diagrams that the distributive laws for sets are correct.

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. 
4. Prove the following two statements.

(a) If $A \subset B$ and $A \subset C$ then $A \subset (B \cap C)$.

(b) If $A \subset C$ and $B \subset C$ then $(A \cup B) \subset C$. 
5. Prove or give a counterexample for each of the following two statements:

(a) If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

(b) If $A \in B$ and $B \in C$ then $A \in C$.
   
   **Hint:** $1 \notin \{\{1\}\}$.  

6. Prove or disprove: for any three sets $A$, $B$, and $C$: $C \setminus (A \cap B) = (C \setminus A) \cap (C \setminus B)$

7. Let $A$ and $B$ be two sets. If $2^A \subseteq 2^B$, what is the relation between $A$ and $B$? (For a set $X$, the set $2^X$ is the set of all the subsets of $X$.) Justify your answer.
8. Let \( a(t) \leq b(t) \leq c(t) \) be the lengths of the three sides of a triangle \( t \) in a non-decreasing order. Define the following sets:

- \( T \): The set of all triangles.
- \( X \): The set of all triangles in which \( a(t) = b(t) \).
- \( Y \): The set of all triangles in which \( b(t) = c(t) \).

Using only set operations on these three set, define the following sets. Justify your answer.

(a) The set of all equilateral triangles (all sides equal).

(b) The set of all isosceles triangles (at least two sides equal).

(c) The set of all scalene triangles (no two sides equal).
9. For two positive integers $m < n$, let $A$ be a set with $m$ elements ($|A| = m$) and $B$ a set of $n$ elements ($|B| = n$). For each of the following sets, give upper and lower bounds on their cardinality.

An upper bound states the maximum possible elements in the set. A lower bound states the minimum possible elements in the set.

Explain your answer.

(a) $A \cap B$

(b) $A \cup B$

(c) $A \setminus B$

(d) $B \setminus A$
10. There are 50 students in a class. 30 students study Math, 25 students study Computer Science, and 15 students study both Math and Computer Science.

   (a) How many students study only Math?  
   (b) How many students study only Computer Science?  
   (c) How many students study neither Math nor Computer Science?  

Explain how you found the answers.