1. Compute \((1001 \mod d)\) for \(d = 2, 3, \ldots, 10\).

Compute \((1001^2 \mod d)\) for \(d = 2, 3, \ldots, 10\).
2. Definition: \( m \) is the **inverse** of \( n \) modulo \( d \) if \( (nm \mod d) = 1 \).

Find the inverse of \( n = 2, 3, \ldots, 10 \) modulo 11 if exist. Explain how you found the inverses.

Find the inverse of \( n = 2, 3, \ldots, 8 \) modulo 9 if exist. Explain how you found the inverses.
3. Euler’s Tution function: \( \varphi(n) \) is the number of positive integers less than \( n \) that are relatively prime to \( n \).
   
   Proposition: \( \varphi(p) = p - 1 \) for any prime number \( p \).
   
   Proposition: \( \varphi(p^k) = p^k - p^{k-1} \) for any positive integer \( k \) and a prime number \( p \).
   
   Proposition: \( \varphi(nm) = \varphi(n)\varphi(m) \) for any two relatively prime \( n \) and \( m \) (gcd\( (n, m) = 1 \)).

Compute \( \varphi(127) \). Justify your answer.

Compute \( \varphi(625) \). Justify your answer.

Compute \( \varphi(713) \). Justify your answer.

Compute \( \varphi(360) \). Justify your answer.
4. Compute \((2^{100} \text{ mod } 3)\), \((2^{100} \text{ mod } 5)\), and \((2^{100} \text{ mod } 7)\). Explain how you got the answer.

Compute \((19^{90} \text{ mod } 31)\). Explain how you got the answer.  
Hint: use Fermat’s Little Theorem

Compute \((47^{61} \text{ mod } 77)\). Explain how you got the answer.  
Hint: use Euler’s Theorem.
5. Definition: gcd(n, m) is the largest positive number that divides both n and m.

Let p ≠ q be two different prime numbers. What is gcd(p, q)? Justify your answer.

Let k and h be two positive integers. What is gcd(2^k, 3^h)? Justify your answer.

Find gcd(1001, 4433) using the Euclid Algorithm. Show the execution of the algorithm.

Find gcd(60, 84, 140).
6. What is the smallest integer \( n \) for which \((n \mod 5) = (n \mod 7) = 1\).

What is the smallest integer \( n \) for which \((n \mod 10) = (n \mod 14) = 1\).

What is the smallest integer \( n \) for which \((n \mod 2) = (n \mod 3) = (n \mod 11) = 1\).

What is the smallest integer \( n \) for which \((n \mod 6) = (n \mod 9) = (n \mod 33) = 1\).

What is the smallest integer \( n \) for which \((n \mod d) = 1\) for all \(2 \leq d \leq 10\). Explain how you found \( n \).