1. Assume a fair coin in which the probability for Head (H) and Tail (T) are both 1/2. Justify your answers.

(a) The fair coin is flipped 4 times.

What is the probability that exactly 2 out of the 4 outcomes are heads?

What is the probability that at least 2 out of the 4 outcomes are heads?

What is the probability that at most 2 out of the 4 outcomes are heads?
(b) The fair coin is flipped 5 times.

What is the probability that exactly 3 out of the 5 outcomes are heads?

What is the probability that at least 3 out of the 5 outcomes are heads?

What is the probability that at most 3 out of the 5 outcomes are heads?
(c) The fair coin is flipped $n \geq 2$ times. Write your answers as a function of $n$.

What is the probability that exactly one out of the $n$ outcomes is head?

What is the probability that at least one out of the $n$ outcomes is head?

What is the probability that at most one out of the $n$ outcomes is head?
2. Assume a biased coin in which the probability for Head (H) is $2/3$ while the probability for Tail (T) is $1/3$. Justify your answers.

   (a) The biased coin is flipped 4 times.

   What is the probability that exactly 2 out of the 4 outcomes are heads?

   What is the probability that at least 2 out of the 4 outcomes are heads?

   What is the probability that at most 2 out of the 4 outcomes are heads?
(b) The biased coin is flipped 5 times.

What is the probability that **exactly** 3 out of the 5 outcomes are heads?

What is the probability that **at least** 3 out of the 5 outcomes are heads?

What is the probability that **at most** 3 out of the 5 outcomes are heads?
(c) The biased coin is flipped $n \geq 2$ times. Write your answers as a function of $n$.

What is the probability that **exactly** one out of the $n$ outcomes is head?

What is the probability that **at least** one out of the $n$ outcomes is head?

What is the probability that **at most** one out of the $n$ outcomes is head?
3. Justify your answers to the following questions.

The fair coin is flipped 4 times. What is the probability that there were no consecutive heads and no consecutive tails?

The fair coin is flipped 5 times. What is the probability that there were no consecutive heads and no consecutive tails?

The fair coin is flipped \( n \geq 1 \) times. What is the probability, as a function of \( n \), that there were no consecutive heads and no consecutive tails?
The biased coin is flipped 4 times. What is the probability that there were no consecutive heads and no consecutive tails?

The biased coin is flipped 5 times. What is the probability that there were no consecutive heads and no consecutive tails?

The biased coin is flipped \( n \geq 1 \) times. What is the probability, as a function of \( n \), that there were no consecutive heads and no consecutive tails?

**Remark:** The answers are different for odd \( n \) and even \( n \).
4. In a regular fair 6-face dice with the numbers 1, 2, 3, 4, 5, 6 on its faces, the probability of throwing any of the 6 numbers is 1/6. Justify your answers.

What is the probability of getting exactly two 6 when three dice are thrown together?

What is the probability of getting exactly two 6 when four dice are thrown together?

What is the probability, as a function of \( n \), of getting exactly two 6 when \( n \geq 2 \) dice are thrown together?
What is the probability of getting at least one 6 when three dice are thrown together?

What is the probability of getting at least one 6 when four dice are thrown together?

What is the probability, as a function of $n$, of getting at least one 6 when $n \geq 1$ dice are thrown together?
5. In a fair 5-face dice with the numbers 1, 2, 3, 4, 5, the probability of throwing any of the 5 numbers is $1/5$. Justify your answers.

What is the probability of getting exactly two 5 when three dice are thrown together?

What is the probability of getting exactly two 5 when four dice are thrown together?

What is the probability, as a function of $n$, of getting exactly two 5 when $n \geq 2$ dice are thrown together?
What is the probability of getting at least one 5 when three dice are thrown together?

What is the probability of getting at least one 5 when four dice are thrown together?

What is the probability, as a function of $n$, of getting at least one 5 when $n \geq 1$ dice are thrown together?
A deck of cards contains 52 cards. There are 4 suits: 13 Black Clubs, 13 Red Diamonds, 13 Red Hearts, and 13 Black Spades. Each suit has one of the following 9 number cards: 2, 3, 4, 5, 6, 7, 8, 9, 10, one of the following 3 face cards: Jack (J), Queen (Q), and King (K), and one Ace (A). A bridge hand has 13 random cards out of the 52 cards.

**Remark:** There are \( \binom{52}{13} \) possible random bridge hands. Define \( x = \binom{52}{13} \). In your answers, you may use \( x \) and define other variables for other large numbers or other large binomial coefficients.

Justify your answers.

(a) What is the probability that all the 13 cards in a bridge hand are of the same suit?

(b) What is the probability that all the 13 cards in a bridge hand are red?
(c) What is the probability that a bridge hand contains only faces and aces?

(d) What is the probability that a bridge hand contains only small cards 2 to 10?

(e) What is the probability that a bridge hand does not contain an ace?
(f) What is the probability that a bridge hand contains all 4 aces?

(g) What is the probability that a bridge hand does not contain two of a kind?
7. A bag contains 16 marbles: 3 Blue marbles, 5 Red (R) marbles, and 8 Green (G) marbles. Justify your answers.

(a) After drawing a marble from the bag the marble is put aside.

What is the probability that 2 drawn marbles are of the same color?

What is the probability that 2 drawn marbles are of different color?

What is the probability that 3 drawn marbles are of the same color?

What is the probability that 3 drawn marbles are of different color?
(b) After drawing a marble from the bag the marble is put back in the bag.

What is the probability that 2 drawn marbles are of the same color?

What is the probability that 2 drawn marbles are of different color?

What is the probability that 3 drawn marbles are of the same color?

What is the probability that 3 drawn marbles are of different color?

What is the probability, as a function of \( n \), that \( n \geq 2 \) drawn marbles are of the same color?