CISC 2210 – Introduction to Discrete Structures

Midterm Exam

Mar 26, 2019

<table>
<thead>
<tr>
<th>Problem</th>
<th>Maximum Points</th>
<th>Your Points</th>
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Grade: 

Id: ..........................................................
1. In the Discrete Structures class there were 3 tests. Call the tests $X$, $Y$, and $Z$.
   • Surprisingly, only 1 student did not get an A on any test while the rest of the students got an A on (aced) at least one test.
   • On the other hand, 4 students aced all three tests!
   • 31 students aced $X$, 42 students aced $Y$, and 37 students, aced $Z$.
   • 13 students aced both $X$ and $Y$, 9 students aced both $X$ and $Z$, and 12 students aced both $Y$ and $Z$.

(a) How many students attended the class? ______
(b) How many students aced only $X$? ______
(c) How many students aced only $Y$? ______
(d) How many students aced only $Z$? ______
(e) How many students aced both $X$ and $Y$ but not $Z$? ______
(f) How many students aced both $X$ and $Z$ but not $Y$? ______
(g) How many students aced both $Y$ and $Z$ but not $X$? ______

Justify all of your answers.
2. Find all the true assignments out of the possible 16 assignments for the 4 variables $x, y, z,$ and $w$ in the following two formulas. You may use truth tables or any other method to find the true assignments. Justify your answers.

$$(x \land y \land z) \lor (x \land y \land w) \lor (x \land z \land w) \lor (y \land z \land w)$$

$$(x \lor y \lor z) \land (x \lor y \lor w) \land (x \lor z \lor w) \land (y \lor z \lor w)$$
3. In all parts of this problem, the goal is to count permutations of the numbers \{1, 2, \ldots, n\} (for \(n = 4, n = 6, \) and general \(n\)) that follow some restrictions.

You **MUST** show how you came up with your answers.

(a) How many permutations of the numbers \{1, 2, 3, 4\} are there?

In how many permutations of the numbers \{1, 2, 3, 4\} the first number is 1?

In how many permutations of the numbers \{1, 2, 3, 4\} the last number is 4?

In how many permutations of the numbers \{1, 2, 3, 4\} the first number is 1 and the last number is 4?

In how many permutations of the numbers \{1, 2, 3, 4\} the first number is not 1?

In how many permutations of the numbers \{1, 2, 3, 4\} the last number is not 4?

In how many permutations of the numbers \{1, 2, 3, 4\} the first number is not 1 and the last number is not 4?
(b) How many permutations of the numbers \{1, 2, 3, 4, 5, 6\} are there?

In how many permutations of the numbers \{1, 2, 3, 4, 5, 6\} the first number is 1?

In how many permutations of the numbers \{1, 2, 3, 4, 5, 6\} the last number is 6?

In how many permutations of the numbers \{1, 2, 3, 4, 5, 6\} the first number is 1 and the last number is 6?

In how many permutations of the numbers \{1, 2, 3, 4, 5, 6\} the first number is not 1?

In how many permutations of the numbers \{1, 2, 3, 4, 5, 6\} the last number is not 6?

In how many permutations of the numbers \{1, 2, 3, 4, 5, 6\} the first number is not 1 and the last number is not 6?
(c) How many permutations of the numbers \{1, 2, \ldots, n\} are there?

In how many permutations of the numbers \{1, 2, \ldots, n\} the first number is 1?

In how many permutations of the numbers \{1, 2, \ldots, n\} the last number is \(n\)?

In how many permutations of the numbers \{1, 2, \ldots, n\} the first number is 1 and the last number is \(n\)?

In how many permutations of the numbers \{1, 2, \ldots, n\} the first number is not 1?

In how many permutations of the numbers \{1, 2, \ldots, n\} the last number is not \(n\)?

In how many permutations of the numbers \{1, 2, \ldots, n\} the first number is not 1 and the last number is not \(n\)?
4. Prove the following identity by induction on $n \geq 1$ or by any other method.

\[ \sum_{i=1}^{n} (4i - 3) = n(2n - 1) \]