Part I: Sets

1. **[12.5 credits]** Do part (a) if the last digit of your ID belongs to \{0, 1, 3, 6, 9\}. Do part (b) if the last digit of your ID belongs to \{2, 4, 5, 7, 8\}. Justify your answers.

   (a) Let \( A \) be a set.
   
   i. What is \( A \setminus \emptyset \)?
   
   ii. What is \( \emptyset \setminus A \)?
   
   iii. What is \( A \triangle \emptyset \)?

   (b) Assume \( A \subseteq B \) for two sets \( A \) and \( B \).
   
   i. What is \( A \setminus B \)?
   
   ii. What is \( A \cap B \)?
   
   iii. What is \( A \cup B \)?

2. **[12.5 credits]** Do part (a) if the last digit of your ID belongs to \{1, 2, 3, 5, 8\}. Do part (b) if the last digit of your ID belongs to \{0, 4, 6, 7, 9\}. Justify your answer.

   (a) Assume that the sets \( A \) and \( B \) have the same cardinality and that \( |A \cup B| = n \) and \( |A \cap B| = m \) for some \( 0 \leq m \leq n \). What are the cardinalities of \( A \) and \( B \)?

   (b) For some \( n \geq 1 \) and two sets \( A \) and \( B \), assume \( |A| = 3n \) and \( |B| = 5n \). If \( |A \cup B| = 3|A \cap B| \), what is \( |A \cup B| \)?

3. **[25 credits]** Assume a class of \( x \) students where \( x \) depends on the 7th digits in your ID.

   - \( x = 36 \) if the 7th digit is 2 or 3.
   - \( x = 48 \) if the 7th digit is 4 or 8.
   - \( x = 60 \) if the 7th digit is 0 or 1 or 5.
   - \( x = 72 \) if the 7th digit is 6 or 9.
   - \( x = 84 \) if the 7th digit is 7.

   The students in this class had three pass/fail quizzes called \( A \), \( B \), and \( C \). Half of the \( x \) students failed \( A \), one third of the \( x \) students failed \( B \), and one fourth of the \( x \) students failed \( C \).

   Justify your answers to the following four questions.

   (a) What is the minimum number of students who could have failed all three quizzes?

   (b) What is the maximum number of students who could have failed all three quizzes?

   (c) What is the minimum number of students who could have passed all three quizzes?

   (d) What is the maximum number of students who could have passed all three quizzes?

   **Bonus credits:** Explain why in the above five options \( x \) must be a multiple of 12.
Part II: Logic

1. [12.5 credits] Do part (a) if the last digit of your ID belongs to \{0, 2, 4, 8, 9\}. Do part (b) if the last digit of your ID belongs to \{1, 3, 5, 6, 7\}. Justify your answers.

   (a) Simplify the following two boolean formulas:
   
   i. \( x \lor (x \land y \land z) \).
   
   ii. \( \neg x \lor (x \lor y \lor z) \).

   (b) Simplify the following two boolean formulas:

   i. \( x \land (\neg x \land \neg y \land \neg z) \).

   ii. \( \neg x \lor (\neg x \land \neg y \land \neg z) \).

2. [12.5 credits] Do parts (a) and (b) if the last digit of your ID belongs to \{0, 1, 2, 4, 7\}. Do parts (c) and (d) if the last digit of your ID belongs to \{3, 5, 6, 8, 9\}. Justify your answer with few sentences.

   (a) Some flowers are colorful. All colorful flowers are beautiful. Hence, all beautiful flowers are colorful. True or False?

   (b) All Roses are flowers. Some flowers are sold in markets. Hence, some Roses are sold in markets. Always True, always False, or could be True and could be False?

   (c) Some students are smart. All smart students got an A on the exam. Hence, all those who got A are smart. True or False?

   (d) All students are smart people. Some smart people earn a lot of money. Hence, some students earn a lot of money. Always True, always False, or could be True and could be False?

3. [25 credits] If the last two digits of your ID are greater than 60 consider the formulas with the \( x \) variables (left side) and if the last two digits of your ID are smaller than 60 consider the formulas with the \( y \) variables (right side).

   Find all True/False assignments for the 3 variables (out of the \( 2^3 = 8 \) possible assignments) that satisfy the following formula. Prove the correctness of your answer.

   \[ x \lor (x_1 \land x_2) \land y \land (y_1 \lor y_2) \]

   **Bonus credits:** For an integer \( n \geq 1 \), determine the number of True/False assignments for the \( n + 1 \) variables (out of the \( 2^{n+1} \) possible assignments) that satisfy the following formula. Justify your answer.

   \[ x \lor (x_1 \land x_2 \land \cdots \land x_n) \land y \land (y_1 \lor y_2 \lor \cdots \lor y_n) \]