1. In how many 3-digit numbers all the digits are odd?

Answer: 125

Solution: There are 5 odd numbers: 1, 3, 5, 7, 9. There are 5 options for each one of the three digits. Therefore, the total number of 3-digit numbers in which all the digits are odd is $5 \cdot 5 \cdot 5 = 125$.

2. In how many 3-digit numbers all the digits are odd and different?

Answer: 60

Solution: There are 5 odd numbers: 1, 3, 5, 7, 9. There are 5 options for the first digit, then 4 options for the second digit, and finally 3 options for the third digit. Therefore, the total number of 3-digit numbers in which all the digits are odd and different is $5 \cdot 4 \cdot 3 = 60$.

3. How many non-empty subsets of \{1, 2, 3, 4, 5, 6, 7, 8, 9\} contain only odd numbers?

Answer: 31

Solution: A non-empty subset of \{1, 2, 3, 4, 5, 6, 7, 8, 9\} that contains only odd numbers is also a subset of \{1, 3, 5, 7, 9\}. Since, a set of size 5 has $2^5 = 32$ subsets among them only one is empty, it follows that \{1, 2, 3, 4, 5, 6, 7, 8, 9\} has 31 non-empty subsets that contain only odd numbers.

4. How many non-empty subsets of \{1, 2, 3, 4, 5, 6, 7, 8, 9\} contain only even numbers?

Answer: 15

Solution: A non-empty subset of \{1, 2, 3, 4, 5, 6, 7, 8, 9\} that contains only even numbers is also a subset of \{2, 4, 6, 8\}. Since, a set of size 4 has $2^4 = 16$ subsets among them only one is empty, it follows that \{1, 2, 3, 4, 5, 6, 7, 8, 9\} has 15 non-empty subsets that contain only even numbers.

5. How many non-empty subsets of \{1, 2, 3, 4, 5, 6, 7, 8, 9\} contain at least one odd number and at least one even number?

Answer: 465

Solution: There are $2^9 - 1 = 511$ non-empty subsets of \{1, 2, 3, 4, 5, 6, 7, 8, 9\}. There are 31 non-empty subsets that contain only odd numbers and there are 15 non-empty subsets that contain only even numbers. As a result, there are $511 - 31 - 15 = 465$ subsets of \{1, 2, 3, 4, 5, 6, 7, 8, 9\} that contain at least one odd number and at least one even number.

6. In how many permutations of 1, 2, 3, 4, and 5, the number 1 appears before the number 5?

Answer: 60

Solution I: Due to symmetry, in half of the $5! = 120$ permutations of \{1, 2, 3, 4, 5\} the number 1 appears before 5 and in half of them 5 appears before 1.

More precisely, let $P$ be the set of all the permutations of \{1, 2, 3, 4, 5\}, let $A \subset P$ be the set of all the permutations in which 1 appears before 5, and let $B \subset P$ the set of all the permutations in which 5 appears before 1. Since, in any permutation either 1 appears before 5 or 5 appears before 1, it follows that $P = A \cup B$ and $A \cap B = \emptyset$. For a permutation $\pi \in A$, let $f(\pi) \in B$ be $\pi$ in which the locations of 1 and 5 are swapped. (For example, $f([31245]) = [35241]$.) The function $f$ is a bijection because $f^{-1}(f(\pi)) = \pi$ for any permutation $\pi \in A$. As a result $|A| = |B|$. Since $P = A \cup B$ and $A \cap B = \emptyset$, it follows that $|A| = |P|/2 = 60$.

Solution II: There are $10 = \binom{5}{2}$ options for the locations of 1 and 5 in the permutation. Since 1 appears before 5, this fixes the locations of both 1 and 5 in the permutation. For each option, there are $6 = 3!$ ways to place the rest of the numbers 2, 3, 4 in the permutation. In total, there are $60 = 10 \cdot 6$ permutations in which 1 appears before 5.