1. How many permutations of the numbers \{1, 2, 3, 4, 5, 6, 7\} are there?
   
   **Answer:** \(7! = 5040\)
   
   **Solution:** For \(n \geq 1\), there are \(n!\) permutations of the numbers \{1, 2, \ldots, n\}.

2. In how many permutations of the numbers \{1, 2, 3, 4, 5, 6, 7\} the first number is 1?
   
   **Answer:** \((7 - 1)! = 6! = 720\)
   
   **Solution:** For \(n \geq 2\), there are \((n - 1)!\) permutations of the numbers \{2, 3, \ldots, n\} to be placed in the last \(n - 1\) positions of a permutation of the numbers \{1, 2, \ldots, n\} in which the first number is 1.

3. In how many permutations of the numbers \{1, 2, 3, 4, 5, 6, 7\} the last number is 7?
   
   **Answer:** \((7 - 1)! = 6! = 720\)
   
   **Solution:** For \(n \geq 2\), there are \((n - 1)!\) permutations of the numbers \{1, 2, \ldots, n - 1\} to be placed in the first \(n - 1\) positions of a permutation of the numbers \{1, 2, \ldots, n\} in which the last number is \(n\).

4. In how many permutations of the numbers \{1, 2, 3, 4, 5, 6, 7\} the first number is 1 and the last number is 7?
   
   **Answer:** \((7 - 2)! = 5! = 120\)
   
   **Solution:** For \(n \geq 3\), there are \((n - 2)!\) permutations of the numbers \{2, 3, \ldots, (n - 1)\} to be placed in positions 2 through \(n - 1\) of a permutation of the numbers \{1, 2, \ldots, n\} in which the first number is 1 and last number is \(n\).

5. In how many permutations of the numbers \{1, 2, 3, 4, 5, 6, 7\} the first number is not 1?
   
   **Answer:** \(7! - (7 - 1)! = 7! - 6! = 5040 - 720 = 4320\)
   
   **Solution:** By parts (1) and (2), there are \(n! - (n - 1)!\) such permutations.

6. In how many permutations of the numbers \{1, 2, 3, 4, 5, 6, 7\} the last number is not 7?
   
   **Answer:** \(7! - (7 - 1)! = 7! - 6! = 5040 - 720 = 4320\)
   
   **Solution:** By parts (1) and (3), there are \(n! - (n - 1)!\) such permutations.

7. In how many permutations of the numbers \{1, 2, 3, 4, 5, 6, 7\} the first number is not 1 and the last number is not 7?
   
   **Answer:** \(7! - 2 \cdot (7 - 1)! + (7 - 2)! = 7! - 2 \cdot 6! + 5! = 5040 - 2 \cdot 720 + 120 = 3720\)
   
   **Solution:** Let \(P\) be the set of all permutations of the numbers \{1, 2, \ldots, n\}, let \(F\) be the set of permutation in \(P\) in which the first number is 1, and let \(L\) be the set of permutation in \(P\) in which the last number is \(n\). The answer is the size of the set

   \[\neg F \cap \neg L\]

   The De Morgan’s laws imply that

   \[\neg F \cap \neg L = \neg (F \cup L)\]

   The inclusion exclusion principle implies that

   \[|F \cup L| = |F| + |L| - |F \cap L|\]

   The solutions to parts (2), (3), and (4) imply that

   \[|F \cup L| = (n - 1)! + (n - 1)! - (n - 2)! = 2 \cdot (n - 1)! - (n - 2)!\]

   Finally, the answer is

   \[|\neg (F \cup L)| = |P \setminus (F \cup L)| = |P| - |(F \cup L)| = n! - 2 \cdot (n - 1)! + (n - 2)!\]