Discrete Math Quiz: Modular Arithmetic

Name and ID: .................................................................

1. Compute \((n \mod d)\) for the following \(n\) and \(d\).
   - \((101 \mod 3) = \) ______
   - \((101 \mod 5) = \) ______
   - \((101 \mod 7) = \) ______
   - \((101^2 \mod 3) = \) ______
   - \((101^2 \mod 5) = \) ______
   - \((101^2 \mod 7) = \) ______

2. Find \((n^{-1} \mod d)\) (inverse of \(n\) modulo \(d\)) for the following \(n\) and \(d\) if exist.
   - \((3^{-1} \mod 7) = \) ______
   - \((4^{-1} \mod 7) = \) ______
   - \((3^{-1} \mod 6) = \) ______
   - \((5^{-1} \mod 6) = \) ______

3. Compute \(\varphi(n)\) for the following \(n\).
   - \(\varphi(17) = \) ______
   - \(\varphi(25) = \) ______
   - \(\varphi(35) = \) ______
   - \(\varphi(36) = \) ______

4. Compute \((n^k \mod d)\) for the following \(n, k,\) and \(d\).
   - \((2^{200} \mod 3) = \) ______
   - \((13^{22} \mod 23) = \) ______
   - \((1001^8 \mod 15) = \) ______

5. Find the greatest common divisors for the following set of numbers.
   - \(\gcd(64, 81) = \) ______
   - \(\gcd(18, 27, 45, 63) = \) ______

6. Answer the following questions.
   - What is the smallest integer \(n > 2\) for which \((n \mod 7) = (n \mod 9) = 2? \) ______
   - What is the smallest integer \(n > 3\) for which \((n \mod 6) = (n \mod 8) = 3? \) ______