Discrete Math Quiz: Modular Arithmetic

Name and ID: .......................................................... .......................................................... ..........................................................

1. Compute \( n \mod d \) for the following \( n \) and \( d \).
   
   (a) \((101 \mod 3) = \) 
   (b) \((101 \mod 5) = \) 
   (c) \((101 \mod 7) = \) 
   (d) \((101^2 \mod 3) = \) 
   (e) \((101^2 \mod 5) = \) 
   (f) \((101^2 \mod 7) = \) 

2. Find \((n^{-1} \mod d)\) (inverse of \( n \) modulo \( d \)) for the following \( n \) and \( d \) if exist.
   
   (a) \((3^{-1} \mod 7) = \) 
   (b) \((4^{-1} \mod 7) = \) 
   (c) \((5^{-1} \mod 6) = \) 
   (d) \((3^{-1} \mod 6) = \) 

3. Compute \( \varphi(n) \) for the following \( n \).
   
   (a) \( \varphi(17) = \) 
   (b) \( \varphi(25) = \) 
   (c) \( \varphi(35) = \) 
   (d) \( \varphi(36) = \) 

4. Compute \((n^k \mod d)\) for the following \( n, k \), and \( d \).
   
   (a) \((2^{200} \mod 3) = \) 
   (b) \((100^{16} \mod 17) = \) 
   (c) \((1001^8 \mod 15) = \) 

5. Find the greatest common divisors for the following set of numbers.
   
   (a) \( \gcd(64, 81) = \) 
   (b) \( \gcd(18, 27, 45, 63) = \) 

6. Find the least common multiply in the first part and answer the question in the second part.
   
   (a) \( \text{lcm}(18, 27, 45) = \) 
   (b) What is the smallest integer \( n > 3 \) for which \((n \mod 6) = (n \mod 8) = 3\)? \( \)