Solutions to Discrete Structures Quiz on Number Bases

1. Write the decimal number \((40)_{10}\) in its binary (base-2) and its ternary (base-3) representations.

\[
40 = 32 + 8 = 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 \implies (40)_{10} = (101000)_2
\]

\[
40 = 27 + 9 + 3 + 1 = 1 \cdot 3^3 + 1 \cdot 3^2 + 1 \cdot 3^1 + 1 \cdot 3^0 \implies (40)_{10} = (1111)_3
\]

2. Write the twelve decimal numbers \((1)_{10}, (2)_{10}, \ldots, (12)_{10}\) in their base-6 representation.

\(1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 20\)

3. Which number is larger \((212)_4\) or \((100110)_2\)?

\[
(212)_4 = 2 \cdot 4^2 + 1 \cdot 4^1 + 2 \cdot 4^0 = 32 + 4 + 2 = 38
\]

\[
(100110)_2 = 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 32 + 4 + 2 = 38
\]

4. In the base-14 system: \(A = 10, B = 11, C = 12\) and \(D = 13\). What is the decimal value of the number \((BAD)_{14}\)?

\[
(BAD)_{14} = 11 \cdot 14^2 + 10 \cdot 14^1 + 13 \cdot 14^0 = 2156 + 140 + 13 = (2309)_{10}
\]

5. What happens to a base-3 number when a 1 is appended at its end?

**Answer:** Let \(x\) be a base-3 number, then

\[(x1)_3 = 3x + 1\]

**Generalization:** What happens to a base-\(b\) number when a \(0 \leq c < b\) is appended at its end?

**Answer:** Let \(x\) be a base-\(b\) number, then

\[(x)c = bx + c\]

6. Which base-2 numbers are multiples of 2?

**Answer:** A base-2 number \(x\) is a multiple of 2 if and only if its last digit is 0.

**Generalization:** Which base-\(b\) numbers are multiples of \(b\)?

**Answer:** A base-\(b\) number \(x\) is a multiple of \(b\) if and only if its last digit is 0. This is true because all the other digits of \(x\) add multiples of \(b^i\) for \(i \geq 1\) and therefore add multiples of \(b\) to the value of \(x\).