Discrete Math

Quiz: Prerequisite

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Grade


1. Identify the five components of the famous formula $e^{\pi i} - 1 = 0$

   (a) The additive identity: _______
   (b) The square root of $-1$: _______
   (c) The multiplicative identity: _______
   (d) The base of the natural logarithm: _______
   (e) The ratio of a circle’s circumference to its diameter: _______

2. Order the following five numbers in an increasing order: $e, \sqrt{2}, 1, \pi, 0$. 

   ___ < ___ < ___ < ___ < ___

3. Let $A$ be the set of all the prime numbers greater than 2 and smaller than 24. Let $B$ be the set of all integers greater than 2 and smaller than 24 that are of the form $4k + 1$ for some integer $k$. Find the following sets:

   (a) $A = $ ________________________________
   (b) $B = $ ________________________________
   (c) $A \cup B = $ ________________________________
   (d) $A \cap B = $ ________________________________

4. Find all possible True and False assignments for the 3 variables $x, y, z$ (out of the possible 8 assignments) that satisfy the following formula:

   $(x \land y) \lor (\bar{x} \land \bar{z})$

Recall that a boolean variable can be either True or False. The AND operation is denoted by $\land$, the OR operation is denoted by $\lor$, and $\bar{v}$ is the negation of the variable $v$. 

5. Expand the following expressions:
   (a) \((x + y)^2 = \) __________
   (b) \((x - y)^2 = \) __________

6. Factor the following expression:
   (a) \(x^2 - y^2 = \) __________

7. (a) Simplify \(x^n \times x^m = \) __________
   (b) Simplify \(x^n \times y^n = \) __________
   (c) If \(\log_a(y) = x\), then \(a^x = \) __________
   (d) If \(\log_a(x) + \log_a(y) = \log_a(z)\), then \(z = \) __________

8. (a) Simplify \(\frac{(n+1)!}{n!} = \) __________
   (b) If \(x < 0\), then \(|x| + |-x| = \) __________

9. Let \(0 \leq x \leq 1\) be a real number and let \(f(x) = \lfloor x \rfloor + \lceil x \rceil\). For which values of \(x\), \(f(x) \neq 1\)?
   Recall that for a real number \(x\), the largest integer that is smaller or equal to \(x\) is denoted by \(\lfloor x \rfloor\) and the smallest integer that is greater or equal to \(x\) is denoted by \(\lceil x \rceil\)

10. Solve the following two linear equations. Find the values of \(x\) and \(y\).
    \[
    \begin{align*}
    x + y & = 20 \\
    2x - 3y & = 5
    \end{align*}
    \]

11. What are the roots of the quadratic equation \(ax^2 + bx + c = 0\)?
12. Let $x = 126$ and $y = 60$.
   (a) What is the Greatest Common Divisor (GCD) of $x$ and $y$? ________________
   (b) What is the Least Common Multiplier (LCM) of $x$ and $y$? ________________

13. Let $T$ be a right-angled triangle with sides $a$, $b$, and $c$ where $c$ is the hypotenuse (the side opposite the right angle). Write $c$ as a function of $a$ and $b$.

   

14. What is the sum of the degrees of all the inner angles of the following geometric shapes?
   (a) Triangle: ________________
   (b) Square: ________________

15. Let $C$ be a circle whose radius is $r$.
   (a) What is the circumference of $C$ as a function of $r$? ________________
   (b) What is the area of $C$ as a function of $r$? ________________

16. When a fair coin is flipped, then both the probabilities of Head (H) and Tail (T) are $1/2$. Three coins are flipped. What is the probability that
   (a) all are H or all are T: ________________
   (b) there is exactly one H: ________________

17. Find the sum of the following sequences as a function of $n$:
   (a) $1 + 2 + 3 + \cdots + n =$ ________________
   (b) $1 + 2 + 4 + 8 + \cdots + 2^n =$ ________________
18. Order the following functions from the slowest to the fastest when \( n \) tends to infinity:

\[
\begin{align*}
n^2 & ; \log(n) & ; n & ; 2^n
\end{align*}
\]

19. Solve the following recursive formulas for \( T(n) \). Express \( T(n) \) as a function of \( n \).

(a) \( n \geq 1 \) is an integer.
\[
\begin{align*}
T(1) &= 1 \\
T(n) &= T(n - 1) + 1
\end{align*}
\]

(b) \( n \geq 0 \) is an integer.
\[
\begin{align*}
T(0) &= 1 \\
T(n) &= 2T(n - 1)
\end{align*}
\]

20. What is the value of \( c \) when each procedure terminates?

(a) \( f(n) \) (* \( n > 0 \) is an integer number *)
\[
\begin{align*}
c &= 0 \\
& \text{for } i = 1 \text{ to } n \text{ do} \\
& \hspace{1em} \text{for } j = 1 \text{ to } n \text{ do} \\
& \hspace{2em} c := c + 1
\end{align*}
\]

(b) \( f(n) \) (* \( n > 0 \) is an integer number *)
\[
\begin{align*}
c &= 1 \\
& \text{for } i = 1 \text{ to } n \text{ do} \\
& \hspace{1em} c := c \times 2
\end{align*}
\]