1. Identify the five components of the famous formula $e^{\pi i} - 1 = 0$
   (a) The additive identity:  
   (b) The square root of $-1$:  
   (c) The multiplicative identity:  
   (d) The base of the natural logarithm:  
   (e) The ratio of a circle’s circumference to its diameter:  

2. Order the following six numbers in an increasing order: $e, \sqrt{2}, \phi, 1, \pi, 0$. By definition, $\phi$ is the golden ratio.
   _ < _ < _ < _ < _ < _

3. Let $A$ be the set of all the prime numbers between 30 and 50. Let $B$ be the set of all odd integers between 30 and 50 that are not of the form $3k + 1$ for some integer $k$. Find the following sets:
   (a) $A =$ ________________________________
   (b) $B =$ ________________________________
   (c) $A \cup B =$ ________________________________
   (d) $A \cap B =$ ________________________________
   (e) $A \setminus B =$ ________________________________
   (f) $B \setminus A =$ ________________________________

4. Find all possible True and False assignments for the 3 variables $x, y, z$ (out of the possible 8 assignments) that satisfy the following formula:

   $(x \lor y) \land (\bar{y} \lor z) \land (\bar{x} \lor \bar{z})$

Recall that a boolean variable can be either True or False. The AND operation is denoted by $\land$, the OR operation is denoted by $\lor$, and $\bar{v}$ is the negation of the variable $v$. 

2
5. Expand the following expressions:

(a) \((x + y)^2 = \) __________________________

(b) \((x - y)^2 = \) __________________________

(c) \((x + y)^3 = \) __________________________

(d) \((x - y)^3 = \) __________________________

(e) \((x + y)^n = \) __________________________

(f) \((x - y)^n = \) __________________________

6. Factor the following expressions:

(a) \(x^2 - y^2 = \) __________________________

(b) \(x^3 - y^3 = \) __________________________

(c) \(x^3 + y^3 = \) __________________________

(d) For any \(n \geq 2\), \(x^n - y^n = \) __________________________

(e) For odd \(n \geq 3\), \(x^n + y^n = \) __________________________

7. (a) Simplify the following expressions:

i. \(x^n \times x^m = \) __________________________

ii. \(x^n \times y^n = \) __________________________

iii. \(\frac{\log_a(x^n)}{\log_a(x)} = \) __________________________

iv. \(2 \log_a(\sqrt{x}) = \) __________________________

(b) Answer the following questions:

i. If \(\log_a(y) = x\), then \(a^x = \) __________________________

ii. If \(\log_a(x) + \log_a(y) = \log_a(z)\), then \(z = \) __________________________

iii. If \(\log_a(x) - \log_a(y) = \log_a(z)\), then \(z = \) __________________________

8. (a) Simplify the following expressions:

i. \(\frac{(n+1)!}{(n-1)!} = \) __________________________

ii. \(\left\lfloor \frac{n}{k} \right\rfloor \times k + (n \mod k) = \) __________________________

(b) Answer the following questions:

i. If \(x > 0\), then \(|x| + |-x| = \) __________________________

ii. If \(x < 0\), then \(|x| + |-x| = \) __________________________
9. Let $0 \leq x \leq 1$ be a real number and let $f(x) = \lfloor x \rfloor + \lceil x \rceil$. For which values of $x$, $f(x) < 1$, $f(x) = 1$, and $f(x) > 1$?

Recall that for a real number $x$, the largest integer that is smaller or equal to $x$ is denoted by $\lfloor x \rfloor$ and the smallest integer that is greater or equal to $x$ is denoted by $\lceil x \rceil$.

10. Solve the following two linear equations. Find the values of $x$ and $y$ as a function of the three constants $a, b, c$.

$$x + y = a$$
$$bx + cy = 0$$

11. What are the roots of the quadratic equation $x^2 + bx = c$?

12. Let $x = 210$ and $y = 225$.

(a) What is the Greatest Common Divisor (GCD) of $x$ and $y$? 
(b) What is the Least Common Multiplier (LCM) of $x$ and $y$?
13. Answer the following two questions:

(a) Let $T$ be a right-angled triangle with sides $a$, $b$, and $c$ where $c$ is the hypotenuse (the side opposite the right angle). Write $c$ as a function of $a$ and $b$.

(b) Let $T$ be a triangle in which one of its side is of length $b$. Let $h$ be the length of the height that is perpendicular to the side $b$. What is the area of the triangle $T$?

14. What is the sum of the degrees of all the inner angles of the following geometric shapes?

(a) Triangle: 

(b) Square: 

(c) Pentagon: 

(d) Hexagon: 

(e) $n$-gon: 

15. Let $C$ be a circle whose radius is $r$ and whose diameter is $d$.

(a) What is the circumference of $C$ as a function of $r$? 

(b) What is the circumference of $C$ as a function of $d$? 

(c) What is the area of $C$ as a function of $r$? 

(d) What is the area of $C$ as a function of $d$? 

16. When a fair coin is flipped, then both the probabilities of Head (H) and Tail (T) are 1/2. Four coins are flipped. What is the probability that

(a) all are H or all are T: 

(b) there is exactly one H: 


17. Find the sum of the following sequences as a function of $n$:

(a) $1 + 2 + 3 + \cdots + n =$

(b) $1 + 2 + 4 + 8 + \cdots + 2^n =$

(c) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n} =$

18. Order the following functions from the slowest to the fastest when $n$ tends to infinity:

$n! ; n^2 ; \log(n) ; n ; 1 ; 2^n ; n^n ; \log \log(n)$
19. Solve the following recursive formulas for $T(n)$. Express $T(n)$ as a function of $n$.

(a) $n \geq 1$ is an integer.
   
   \[
   T(1) = 1  \\
   T(n) = T(n - 1) + 1
   \]

(b) $n \geq 1$ is an integer.
   
   \[
   T(1) = 1  \\
   T(n) = T(n - 1) + n
   \]

(c) $n \geq 0$ is an integer.
   
   \[
   T(0) = 1  \\
   T(n) = 2T(n - 1)
   \]

(d) $n \geq 1$ is a power of 2 integer.
   
   \[
   T(1) = 0  \\
   T(n) = T(n/2) + 1
   \]
What is the value of $c$ when each procedure terminates?

(a) $f(n)$ (* $n \geq 1$ is an integer *)
\[
c = 0 \\
\text{for } i = 1 \text{ to } n \text{ do} \\
\quad \text{for } j = 1 \text{ to } n \text{ do} \\
\quad \quad c := c + 1
\]

(b) $f(n)$ (* $n \geq 1$ is an integer *)
\[
c = 0 \\
\text{for } i = 1 \text{ to } n \text{ do} \\
\quad \text{for } j = i \text{ to } n \text{ do} \\
\quad \quad c := c + 1
\]

(c) $f(n)$ (* $n \geq 1$ is an integer *)
\[
c = 1 \\
\text{for } i = 1 \text{ to } n \text{ do} \\
\quad c := c \times 2
\]

(d) $f(n)$ (* $n \geq 1$ is a power of 2 integer *)
\[
c = 0 \\
\text{while } n > 1 \text{ do} \\
\quad n := n/2 \\
\quad c := c + 1
\]