Discrete Math

Quiz: Prerequisite

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1. Identify the five components of the famous formula $e^{\pi i} - 1 = 0$

(a) The additive identity: _____
(b) The square root of $-1$: _____
(c) The multiplicative identity: _____
(d) The base of the natural logarithm: _____
(e) The ratio of a circle’s circumference to its diameter: _____

2. Order the 5 numbers $\sqrt{2}, \pi, 0, e, 1$ in an increasing order:

___ < ___ < ___ < ___ < ___

3. Let \( A \) be the set of all the prime numbers greater than 10 and smaller than 32. Let \( B \) be the set of all integers greater than 10 and smaller than 32 that are of the form $4k - 1$ for some integer $k$. Find the following sets:

(a) \( A = \) ________________________________
(b) \( B = \) ________________________________
(c) \( A \cup B = \) ________________________________
(d) \( A \cap B = \) ________________________________

4. Working with formulas and expressions.

(a) Expand the expression \((x - y)^2 = \) ________________________________
(b) Factor the expression \(y^2 - x^2 = \) ________________________________
(c) Simplify the expression \(x^k \times x^h = \) ________________________________
(d) Answer the question: If \( \log_a(y) = x \), then \( a^x = \) ________________________________
(e) Simplify the expression \( \frac{n!}{(n-1)!} = \) ________________________________

5. Let \( x = 15 \) and \( y = 10 \).

(a) What is the Greatest Common Divisor (GCD) of \( x \) and \( y \)? ________
(b) What is the Least Common Multiplier (LCM) of \( x \) and \( y \)? ________

6. When a fair coin is flipped, then both the probabilities of Head (H) and Tail (T) are 1/2. Three coins are flipped. What is the probability \( P \) that exactly one of them is \( H \)? \( P = \) ________________
(a) Let $T$ be a right-angled triangle with sides $a$, $b$, and $c$ where $c$ is the hypotenuse (the side opposite the right angle).
Write $a$ as a function of $b$ and $c$: __________________________
(b) What is the sum of the degrees of all the four inner angles of any rectangle? ________________________________
(c) What is the area of a circle $C$ as a function of its radius $r$? $C =$ __________

8. Find the values of $x$ and $y$ in the linear equations:

\[
\begin{align*}
    x - y &= 2 \\
    2x + 3y &= 19
\end{align*}
\]

$x =$ _____
y =$ _____

9. Find the sum $S$ of the sequence $1 + 2 + 4 + 8 + \cdots + 2^n$ as a function of $n$.
$S =$ _______________________

10. Order the four functions $2^n$, $\log(n)$, $n$, and $n^2$ by their growth from the slowest to the fastest when $n$ tends to infinity:

_____ $<$ _____ $<$ _____ $<$ _____

11. Express the following recursive formula as a function of an integer $n \geq 1$:
$T(n) =$ __________________________

$T(1) = 1$
$T(n) = T(n - 1) + 1$

12. What is the value of $c$ when each procedure terminates? $c =$ ________

\[
f(n) \quad (\ast \ n > 0 \ \text{is an integer number} \ \ast) \\
c = 0 \\
\text{for } i = 1 \text{ to } n \text{ do}
\quad \text{for } j = 1 \text{ to } n \text{ do}
\qquad c := c + 1
\]