Discrete Structures: Probability

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Introduction

**A song**

[https://www.youtube.com/watch?v=au8hf0-A27s](https://www.youtube.com/watch?v=au8hf0-A27s)

**Likelihood**

[https://www.youtube.com/watch?v=QpfMwA0z_1Y](https://www.youtube.com/watch?v=QpfMwA0z_1Y)
Simple Demonstrating Examples

Probability and counting: guessing in T/F exams
  https://www.youtube.com/watch?v=3V2omKRX9gc

A problem with three methods to solve it
  https://youtu.be/fyv1i2YbNt0

A dice problem
  https://www.youtube.com/watch?v=Kgudt4PXs28&feature=youtu.be

Dice Example: Independence addition and complement
  https://www.youtube.com/watch?v=dqNvtYWmSrY&list=PL482E6C6B5F0A30E7&index=7&t=0s

Marble Example: addition vs. multiplication
  https://www.youtube.com/watch?v=lA7rprQecV8&list=PL482E6C6B5F0A30E7&index=6&t=0s
Law of Large Numbers & Probabilities via Simulations

Introducing the concept
https://www.youtube.com/watch?v=MntX3zWNWec

Experimental versus theoretical probability
https://www.youtube.com/watch?v=Nos-xOCpQgg&feature=youtu.be

Collecting six coupons via simulations
https://www.youtube.com/watch?v=2AVLfSRpmfg&feature=youtu.be
Conditional Probability

The formula for two events with a proof by example

https://www.youtube.com/watch?v=H02B3aMNKzE&list=PL482E6C6B5F0A30E7&index=2&t=0s

Statistics example: independence and conditional probability

https://youtu.be/pIfpHdGVwLU

Coin example: conditional probability

https://youtu.be/Zxm4Xxvzohk

Conditional probability via false positive and false negative

https://youtu.be/hxEdXUB_IdQ

Bayes theorem, the geometry of changing beliefs

https://www.youtube.com/watch?v=HZGCoVF3YvM
Two Children One of Them is a Boy Like Problems

The Original Problem
- https://www.youtube.com/watch?v=MDzbD2Ay5b4

Variations
- Three red-blue cards: https://www.youtube.com/watch?v=1IFM1ngwEb0
- 2 Aces and 2 twos: https://www.youtube.com/watch?v=TGukPgViEkg
- A dice version: https://www.youtube.com/watch?v=lTJydnfGyyk

The Frogs Story
- Wrong: https://www.youtube.com/watch?v=cpwSGsb-rTs&v1=en
- Correct: https://www.youtube.com/watch?v=go3xtDdsNQM
More About Statistics

Biases in Statistics

- Examples of bias in surveys
  https://cdn.kastatic.org/ka-youtube-converted/r1R00l8Z5lg.mp4/r1R00l8Z5lg.mp4#t=8
- Example of undercoverage introducing bias
  https://cdn.kastatic.org/ka-youtube-converted/D9f7dCMxu2Y.mp4/D9f7dCMxu2Y.mp4#t=0
- Sources of Bias
  https://www.youtube.com/watch?v=j8sOuCo0ybY

Application

- Why do airlines sell too many tickets?
  https://youtu.be/ZFNstNKgEDI
Intuition Or Not?

Monty Hall
- https://youtu.be/_X5erR9LKUs
- https://youtu.be/4Lb-6rxZxx0

The birthday “paradox”
- https://youtu.be/KtT_cgMzHx8

The false positive “paradox”
- https://youtu.be/1csFTDXXULY
Problems With “Counter Intuitive” Answers

Relatively Easier Problems

- Random walkers meeting on a grid
  - https://www.youtube.com/watch?v=F_kt51Qj1RI
- Guessing randomly on A matching test
  - https://www.youtube.com/watch?v=h2VMMlpEtRU

Relatively Harder Problems

- What is the chance twin brothers are identical?
  - https://www.youtube.com/watch?v=xBPMoavS5Hs
- HH needs 6 and HT needs 4 flips via markov chains
  - https://www.youtube.com/watch?v=IAiNqQi30-Y
  - https://www.youtube.com/watch?v=-mxW3jDlGk
- Conditional probability with boys and girls
  - https://www.youtube.com/watch?v=UQnBGmlaLjY
Paradoxes

The Simpson’s paradox

- https://youtu.be/sxYrzzy3cq8
- https://www.youtube.com/watch?v=ebEkn-BiW5k
- https://www.youtube.com/watch?v=E_ME4P9fQbo&vl=en
- https://www.youtube.com/watch?v=cOQFSmj1jv0
- https://www.youtube.com/watch?v=ZDinnCwP3dg

Bertrand’s paradox

- https://youtu.be/xy6xEhbGa0
Beyond Basic Probability

Fermat needs 2 wins Euler needs 3 wins
https://www.youtube.com/watch?v=C_nV3cVNjog

Weird dice with the same distribution
https://www.youtube.com/watch?v=kG4Fh_PrN-E

The reason casinos always win meet the law of large numbers
https://www.youtube.com/watch?v=RXY-WN0ahiw

An alien extinction riddle
https://www.youtube.com/watch?v=A5-Q2GdD5xw