Discrete Structures: Probability

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Basic Concepts

- **A song**
  - https://www.youtube.com/watch?v=au8hf0-A27s

- **Likelihood**
  - https://www.youtube.com/watch?v=QpfMwA0z_1Y

- **Probability and counting: guessing in a T/F exam**
  - https://www.youtube.com/watch?v=3V2omKRX9gc

- **10 minute introduction**
  - https://www.youtube.com/watch?v=WqTioYM0L7I
Simple Demonstrating Examples

- **A basic problem with three solutions:**
  - [https://www.youtube.com/watch?v=fyvli2YbNt0](https://www.youtube.com/watch?v=fyvli2YbNt0)

- **A good dice problem:**

- **Dice Example: Independence addition and complement**
  - [https://www.youtube.com/watch?v=dqNvtYWmSrY&list=PL482E6C6B5F0A30E7&index=7&t=0s](https://www.youtube.com/watch?v=dqNvtYWmSrY&list=PL482E6C6B5F0A30E7&index=7&t=0s)

- **Marble Example: addition vs. multiplication**
  - [https://www.youtube.com/watch?v=lA7rprQecV8&list=PL482E6C6B5F0A30E7&index=6&t=0s](https://www.youtube.com/watch?v=lA7rprQecV8&list=PL482E6C6B5F0A30E7&index=6&t=0s)

- **Marble example: with or without replacement**
  - [https://www.youtube.com/watch?v=ewdsEDuXMVM&list=PL482E6C6B5F0A30E7&index=3&t=0s](https://www.youtube.com/watch?v=ewdsEDuXMVM&list=PL482E6C6B5F0A30E7&index=3&t=0s)

- **Example with probability tree**
  - [https://www.youtube.com/watch?v=OBbHws_BAGs&list=PL482E6C6B5F0A30E7&index=5&t=0s](https://www.youtube.com/watch?v=OBbHws_BAGs&list=PL482E6C6B5F0A30E7&index=5&t=0s)
Law of Large Numbers and Finding Probabilities via Simulations

- Introducing the concept
  - https://www.youtube.com/watch?v=MntX3zWNWeC

- Experimental versus theoretical probability simulation

- Collecting 6 coupons via simulations:
Conditional Probability

- The formula for 2 events with a proof by example
  - https://www.youtube.com/watch?v=H02B3aMNKzE&list=PL482E6C6B5F0A30E7&index=2&t=0s

- Statistics example: independence and conditional probability

- Coin example: conditional probability

- Conditional probability via false positive and false negative
Two Children One of Them is a Boy Like Problems

The Original Problem

- [https://www.youtube.com/watch?v=MDzbD2Ay5b4](https://www.youtube.com/watch?v=MDzbD2Ay5b4)

Variations

- Three red-blue cards: [https://www.youtube.com/watch?v=1IFM1ngwEb0](https://www.youtube.com/watch?v=1IFM1ngwEb0)
- 2 Aces and 2 twos: [https://www.youtube.com/watch?v=TGukPgViEkg](https://www.youtube.com/watch?v=TGukPgViEkg)
- A dice version: [https://www.youtube.com/watch?v=lTJydnfGyyk](https://www.youtube.com/watch?v=lTJydnfGyyk)

The Frogs Story

- Wrong: [https://www.youtube.com/watch?v=cpwSGsb-rTs&vl=en](https://www.youtube.com/watch?v=cpwSGsb-rTs&vl=en)
- Correct: [https://www.youtube.com/watch?v=go3xtDdsNQM](https://www.youtube.com/watch?v=go3xtDdsNQM)
Applications

- Why do airlines sell too many tickets?
  
Intuition Or Not?

- Monty hall:
  

- The birthday “paradox”:
  

- The false positive “paradox”:
  
  https://ed.ted.com/lessons/can-you-solve-the-false-positive-riddle-alex-gendler
Problems With “Counter Intuitive” Answers

Relatively Easier Problems
- Random walkers meeting on a grid
  - https://www.youtube.com/watch?v=F_kt51Qj1RI
- Guessing randomly on a matching test
  - https://www.youtube.com/watch?v=h2VMM1pEtRU

Relatively Harder Problems
- What is the chance twin brothers are identical?
  - https://www.youtube.com/watch?v=xBPMoavS5Hs
- HH needs 6 and HT needs 4 flips via markov chains
  - https://www.youtube.com/watch?v=IAiNqQi30-Y
  - https://www.youtube.com/watch?v=--mxW3jDlGk
- Conditional probability with boys and girls
  - https://www.youtube.com/watch?v=UQnBGmlaLjY
Paradoxes

The Simpson’s paradox

- https://www.youtube.com/watch?v=ebEkn-BiW5k
- https://www.youtube.com/watch?v=E_ME4P9fQbo&vl=en
- https://www.youtube.com/watch?v=cOQFSmj1jv0
- https://www.youtube.com/watch?v=ZDinnCwP3dg

Bertrand’s paradox

Beyond Basic Probability

- Fermet needs 2 wins Euler needs 3 wins
  - https://www.youtube.com/watch?v=C_nV3cVNjog

- Weird dice with the same distribution:
  - https://www.youtube.com/watch?v=kG4Fh_PrN-E

- The reason casinos always win meet the law of large numbers:
  - https://www.youtube.com/watch?v=RXY-WN0ahiw

- An alien extinction riddle:
  - https://www.youtube.com/watch?v=A5-Q2GdD5xw