Discrete Math Quiz: Number Theory

Name and ID: .................................................................

1. Compute \((n \mod d)\) for the following \(n\) and \(d\).
   
   (a) \((29 \mod 3) = \) ______
   
   (b) \((29 \mod 5) = \) ______
   
   (c) \((29 \mod 7) = \) ______
   
   (d) \((29^2 \mod 3) = \) ______
   
   (e) \((29^2 \mod 5) = \) ______
   
   (f) \((29^2 \mod 7) = \) ______

2. Find, if it exists, \((n^{-1} \mod d)\) (inverse of \(n\) modulo \(d\)) for the following \(n\) and \(d\).
   
   (a) \((3^{-1} \mod 5) = \) ______
   
   (b) \((4^{-1} \mod 5) = \) ______
   
   (c) \((3^{-1} \mod 4) = \) ______
   
   (d) \((2^{-1} \mod 4) = \) ______

3. Compute \(\varphi(n)\) for the following \(n\).
   
   (a) \(\varphi(11) = \) ______
   
   (b) \(\varphi(9) = \) ______
   
   (c) \(\varphi(10) = \) ______
   
   (d) \(\varphi(36) = \) ______

4. Compute \((n^k \mod d)\) for the following \(n\), \(k\), and \(d\).
   
   (a) \((2^{20} \mod 3) = \) ______
   
   (b) \((11^{12} \mod 13) = \) ______
   
   (c) \((1001^2 \mod 6) = \) ______

5. Find the greatest common divisors for the following set of numbers.
   
   (a) \(\gcd(16, 27) = \) ______
   
   (b) \(\gcd(14, 21, 35, 42) = \) ______

6. Find the least common multiply in the first part and answer the question in the second part.
   
   (a) \(\text{lcm}(14, 21, 35) = \) ______
   
   (b) What is the smallest integer \(n > 2\) for which \((n \mod 4) = (n \mod 6) = 2\)? \(\) ______