1. Compute \((n \mod d)\) for the following \(n\) and \(d\).
   - \((101 \mod 3) = 2\) because \(101 = 33 \cdot 3 + 2\)
   - \((101 \mod 5) = 1\) because \(101 = 20 \cdot 5 + 1\)
   - \((101 \mod 7) = 3\) because \(101 = 14 \cdot 7 + 3\)
   - \((101^2 \mod 3) = 1\) because \((101^2 \mod 3) = ((101 \mod 3)^2 \mod 3) = (2^2 \mod 3) = (4 \mod 3) = 1\)
   - \((101^2 \mod 5) = 1\) because \((101^2 \mod 5) = ((101 \mod 5)^2 \mod 5) = (1^2 \mod 5) = (1 \mod 5) = 1\)
   - \((101^2 \mod 7) = 2\) because \((101^2 \mod 7) = ((101 \mod 7)^2 \mod 7) = (3^2 \mod 7) = (9 \mod 7) = 2\)

2. Find, if it exists, \((n^{-1} \mod d)\) (inverse of \(n\) modulo \(d\)) for the following \(n\) and \(d\).
   - \((3^{-1} \mod 7) = 5\) because \(3 \cdot 5 = 15 = 2 \cdot 7 + 1\)
   - \((4^{-1} \mod 7) = 2\) because \(4 \cdot 2 = 8 = 1 \cdot 7 + 1\)
   - \((5^{-1} \mod 6) = 5\) because \(5 \cdot 5 = 25 = 4 \cdot 6 + 1\)
   - \((3^{-1} \mod 6)\) does not exist because \((n \cdot 3 \mod 6)\) is either 0 or 3 for any integer \(n\).

3. Compute \(\varphi(n)\) for the following \(n\).
   - \(\varphi(17) = 17 - 1 = 16\)
   - \(\varphi(25) = \varphi(5^2) = 5^2 - 5^1 = 25 - 5 = 20\)
   - \(\varphi(35) = \varphi(7)\varphi(5) = (7 - 1)(5 - 1) = 6 \cdot 4 = 24\)
   - \(\varphi(36) = \varphi(3^2 \cdot 2^2) = \varphi(3^2)\varphi(2^2) = (3^2 - 3^1)(2^2 - 2^2) = 6 \cdot 2 = 12\)

4. Compute \((n^k \mod d)\) for the following \(n, k,\) and \(d\).
   - \((2^{200} \mod 3) = ((2^2)^{100} \mod 3) = (4^{100} \mod 3) = ((4 \mod 3)^{100} \mod 3) = (1^{100} \mod 3) = 1\)
   - \((100^{16} \mod 17) = 1\) by Fermat’s little Theorem because 17 is a prime number that is not a divisor of 100.
   - \((1001^8 \mod 15) = 1\) by Euler’s Theorem because \(\gcd(1001, 15) = 1\) and \(\varphi(15) = \varphi(5)\varphi(3) = (5 - 1)(3 - 1) = 4 \cdot 2 = 8\)

5. Find the greatest common divisors for the following set of numbers.
   - \(\gcd(64, 81) = 1\) because the only divisors of 64 are powers of 2 while the only divisors of 81 are powers of 3.
   - \(\gcd(18, 27, 45, 63) = 9\) because 9 divides these four numbers, 18 does not divide the other three numbers, and any number between 9 and 18 does not divide 18.

6. Find the least common multiply in the first part and answer the question in the second part.
   - \(\text{lcm}(18, 27, 45) = 9 \cdot 2 \cdot 3 \cdot 5 = 270\) because 9 divides 18, 27, and 45, and the other prime factors of these numbers are 2, 3, and 5.
   - \(\text{lcm}(6, 8) + 3 = 24 + 3 = 27\) is the smallest integer \(n > 3\) for which \((n \mod 6) = (n \mod 8) = 3\).