Discrete Math Quiz: Number Theory

Name and ID: .................................................................

1. Compute \((n \mod d)\) for the following \(n\) and \(d\).
   (a) \((92 \mod 3) = \) ______
   (b) \((92 \mod 5) = \) ______
   (c) \((92 \mod 7) = \) ______
   (d) \((92^2 \mod 3) = \) ______
   (e) \((92^2 \mod 5) = \) ______
   (f) \((92^2 \mod 7) = \) ______

2. Find, if it exists, \((n^{-1} \mod d)\) (inverse of \(n\) modulo \(d\)) for the following \(n\) and \(d\).
   (a) \((2^{-1} \mod 7) = \) ______
   (b) \((5^{-1} \mod 7) = \) ______
   (c) \((3^{-1} \mod 8) = \) ______
   (d) \((4^{-1} \mod 8) = \) ______

3. Compute \(\varphi(n)\) for the following \(n\).
   (a) \(\varphi(19) = \) ______
   (b) \(\varphi(49) = \) ______
   (c) \(\varphi(21) = \) ______
   (d) \(\varphi(48) = \) ______

4. Compute \((n^k \mod d)\) for the following \(n, k,\) and \(d\).
   (a) \((2^{100} \mod 3) = \) ______
   (b) \((100^{18} \mod 19) = \) ______
   (c) \((901^8 \mod 15) = \) ______

5. Find the greatest common divisors for the following set of numbers.
   (a) \(\gcd(32, 25) = \) ______
   (b) \(\gcd(22, 33, 55, 77) = \) ______

6. Find the least common multiply in the first part and answer the question in the second part.
   (a) \(\text{lcm}(22, 33, 55) = \) ______
   (b) What is the smallest integer \(n > 3\) for which \((n \mod 6) = (n \mod 9) = 3\)?   ______