1. Identify the five components $e$, $\pi$, $i$, $1$, and $0$ that appear in the famous formula: $e^{\pi i} + 1 = 0$.

   (a) The additive identity: _______
   (b) The square root of $-1$: _______
   (c) The multiplicative identity: _______
   (d) The base of the natural logarithm: _______
   (e) The ratio of a circle’s circumference to its diameter: _______

2. Order the following six numbers in an increasing order: $e$, $\sqrt{2}$, $\phi$, $1$, $\pi$, $0$ ($\phi$ is the golden ratio).

   ____ < ____ < ____ < ____ < ____ < ____

3. Let $A$ be the set of all the prime numbers between 30 and 50. Let $B$ be the set of all odd integers between 30 and 50 that are not of the form $3k + 1$ for some integer $k$. Find the following sets:

   (a) $A = $ __________________________
   (b) $B = $ __________________________
   (c) $A \cup B = $ __________________________
   (d) $A \cap B = $ __________________________

4. (a) Expand the following expressions:

   i. $(x + y)^2 =$ __________________________
   ii. $(x - y)^2 =$ __________________________
   iii. $(x + y)^3 =$ __________________________
   iv. $(x - y)^3 =$ __________________________
   v. $(x + y)^n =$ __________________________
   vi. $(x - y)^n =$ __________________________

   (b) Factor the following expressions:

   i. $x^2 - y^2 =$ __________________________
   ii. $x^3 - y^3 =$ __________________________
   iii. $x^3 + y^3 =$ __________________________
   iv. For any $n \geq 2$, $x^n - y^n =$ __________________________
   v. For odd $n \geq 3$, $x^n + y^n =$ __________________________
5. (a) Simplify the following expressions:
   i. \( x^n \times x^m = \) 
   ii. \( x^n \times y^n = \) 
   iii. \( \frac{\log_a(x^n)}{\log_a(x)} = \) 
   iv. \( 2 \log_a(\sqrt{x}) = \)

(b) Answer the following questions:
   i. If \( \log_a(y) = x \), then \( a^x = \) 
   ii. If \( \log_a(x) + \log_a(y) = \log_a(z) \), then \( z = \) 
   iii. If \( \log_a(x) - \log_a(y) = \log_a(z) \), then \( z = \)

6. (a) \( 4! = \) 
   (b) If \( n! = 120 \) then \( n = \) 
   (c) If \( n! = 720 \) then \( n = \) 
   (d) Simplify \( \frac{(n+1)!}{(n-1)!} = \) 

7. (a) Solve the following two linear equations. Find the values of \( x \) and \( y \) as a function of the three constants \( a, b, c \).
   \[ x + y = a \]
   \[ bx + cy = 0 \]
   \( x = \)
   \( y = \)

   (b) What are the roots of the quadratic equation \( x^2 + bx = c \)?
   \( x_1 = \)
   \( x_2 = \)

8. When a fair coin is flipped, then both the probabilities of Head (H) and Tail (T) are 1/2. Four fair coins are flipped. What is the probability that
   (a) all the coins are the same (either all show T or all show H): 
   (b) exactly one coin shows H while the other three coins show T: 
   (c) exactly two coins show T and two coins show H:
9. (a) Answer the following two questions:
   i. Let $T$ be a right-angled triangle with sides $a$, $b$, and $c$ where $c$ is the hypotenuse (the side opposite the right angle). Write $c$ as a function of $a$ and $b$.

   ii. Let $T$ be a triangle in which one of its side is of length $b$. Let $h$ be the length of the height that is perpendicular to the side $b$. What is the area of the triangle $T$?

(b) What is the sum of the degrees of all the inner angles of the following geometric shapes?
   i. Triangle: ________________________________
   ii. Square: ________________________________
   iii. Pentagon: ________________________________
   iv. Hexagon: ________________________________
   v. $n$-gon: ________________________________

(c) Let $C$ be a circle whose radius is $r$ and whose diameter is $d$.
   i. What is the circumference of $C$ as a function of $r$? ________________________________
   ii. What is the circumference of $C$ as a function of $d$? ________________________________
   iii. What is the area of $C$ as a function of $r$? ________________________________
   iv. What is the area of $C$ as a function of $d$? ________________________________

10. What is the value of $c$ when each procedure terminates?

   (a) $f(n)$ (* $n \geq 1$ is an integer *)
      
      $c = 0$
      
      for $i = 1$ to $n$
      
      for $j = 1$ to $n$
      
      $c := c + 1$

   $c = ________________________________$

   (b) $f(n)$ (* $n \geq 1$ is an integer *)
      
      $c = 0$
      
      for $i = 1$ to $n$
      
      for $j = i$ to $n$
      
      $c := c + 1$

   $c = ________________________________$

   (c) $f(n)$ (* $n \geq 1$ is an integer *)
      
      $c = 1$
      
      for $i = 1$ to $n$
      
      $c := c \times 2$

   $c = ________________________________$

   (d) $f(n)$ (* $n \geq 1$ is a power of 2 integer *)
      
      $c = 0$
      
      while $n > 1$
      
      $n := n/2$
      
      $c := c + 1$

   $c = ________________________________$
11. Express the sum of the following sequences as a function of \( n \):

(a) \( 1 + 2 + 3 + \cdots + n = \) ________________________________

(b) \( 1 + 2 + 4 + 8 + \cdots + 2^n = \) ________________________________

(c) \( \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n} = \) ________________________________

12. Order the following 8 functions by their growth from the slowest to the fastest when \( n \) tends to infinity:

\[ n! ; \ n^2 ; \ \log(n) ; \ n ; \ 1 ; \ 2^n ; \ n^n ; \ \log \log(n) \]

___ < ___ < ___ < ___ < ___ < ___ < ___ < ___