1. The five components of the famous formula \( e^{\pi i} + 1 = 0 \) are:
   (a) The additive identity: 0
   (b) The square root of \(-1\): \( i \)
   (c) The multiplicative identity: 1
   (d) The base of the natural logarithm: \( e \)
   (e) The ratio of a circle’s circumference to its diameter: \( \pi \)

2. \( 0 < 1 < \sqrt{2} = 1.414... < \phi = 1.618... < e = 2.718... < \pi = 3.141... \)

3. Let \( A \) be the set of all the prime numbers between 30 and 50. Let \( B \) be the set of all odd integers between 30 and 50 that are not of the form \( 3k + 1 \) for some integer \( k \).
   (a) \( A = \{31, 37, 41, 43, 47\} \)
   (b) \( B = \{33, 35, 39, 41, 45, 47\} \)
   (c) \( A \cup B = \{31, 33, 35, 37, 39, 41, 43, 45, 47\} \)
   (d) \( A \cap B = \{41, 47\} \)

4. (a) i. \( (x + y)^2 = x^2 + 2xy + y^2 \)
    ii. \( (x - y)^2 = x^2 - 2xy + y^2 \)
    iii. \( (x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 \)
    iv. \( (x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3 \)
    v. \( (x + y)^n = \sum_{i=0}^{n} (\binom{n}{i})x^{n-i}y^i = x^n + nx^{n-1}y + \ldots + nxy^{n-1} + y^n \)
    vi. \( (x - y)^n = \sum_{i=0}^{n} (-1)^i(\binom{n}{i})x^{n-i}y^i = x^n - nx^{n-1}y + \ldots \pm nxy^{n-1} \pm y^n \)
   (b) i. \( x^2 - y^2 = (x - y)(x + y) \)
    ii. \( x^3 - y^3 = (x - y)(x^2 + xy + y^2) \)
    iii. \( x^3 + y^3 = (x + y)(x^2 - xy + y^2) \)
    iv. \( x^n - y^n = (x - y)(x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \ldots + x^2y^{n-3} + xy^{n-2} + y^{n-1}) \)
    v. \( x^n + y^n = (x + y)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 - \ldots + x^2y^{n-3} - xy^{n-2} + y^{n-1}) \)
    only for odd \( n \).
5. (a) i. \( x^n \times x^m = x^{n+m} \)
   
   ii. \( x^n \times y^n = (xy)^n \)

   iii. \( \frac{\log_a(x^n)}{\log_a(x)} = n \)

   iv. \( 2\log_a(\sqrt{x}) = \log_a(x) \)

   (b) i. If \( \log_a(y) = x \), then \( a^x = y \)

   ii. If \( \log_a(x) + \log_a(y) = \log_a(z) \), then \( z = xy \)

   iii. If \( \log_a(x) - \log_a(y) = \log_a(z) \), then \( z = x/y \)

6. (a) \( 4! = 24 \)
   
   (b) \( 5! = 120 \)

   (c) \( 6! = 720 \)

   (d) \( \frac{(n+1)!}{(n-1)!} = n(n + 1) \)

7. (a) For the following 2 linear equations

   \[
   \begin{align*}
   x + y &= a \\
   bx + cy &= 0
   \end{align*}
   \]

   The values of \( x \) and \( y \) as a function of the three constants \( a, b, c \) are:

   \[
   x = \frac{ac}{c - b} \quad \text{and} \quad y = \frac{-ab}{c - b}
   \]

(b) The roots of the quadratic equation \( x^2 + bx = c \) (equivalently, \( x^2 + bx - c = 0 \)) are

   \[
   x_1 = \frac{-b + \sqrt{b^2 + 4c}}{2} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 + 4c}}{2}
   \]

8. Four fair coins are flipped. The 16 possible outcomes are

   • HHHH, HHHT, HHTH, HHTT, HTHH, HTTH, HTHT, HTTT, Thhh, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT

   (a) The probability that all the coins show H or all the coins show T is \( \frac{2}{16} = \frac{1}{8} \)

   • HHHH and TTTT

   (b) The probability that exactly one coin shows H while the other three coins show T is \( \frac{4}{16} = \frac{1}{4} \)

   • HTTT, THTT, TTHT, and TTTH

   (c) The probability that exactly two coins show T and two coins show H is \( \frac{6}{16} = \frac{3}{8} \)

   • TTHH, THHT, HTHT, HTTH, HTHT, and HHTT
9. (a) i. Let $T$ be a right-angled triangle with sides $a$, $b$, and $c$ where $c$ is the hypotenuse (the side opposite the right angle). Then
$$c^2 = a^2 + b^2 \implies c = \sqrt{a^2 + b^2}$$

ii. Let $T$ be a triangle in which one of its side is of length $b$. Let $h$ be the length of the height that is perpendicular to the side $b$. Then the area of the triangle $T$ is
$$\frac{b \cdot h}{2}$$

(b) The sum of the degrees of all the inner angles of the following geometric shapes is
i. Triangle: $180$
ii. Square: $360$
iii. Pentagon: $540$
iv. Hexagon: $720$
v. $n$-gon: $180(n - 2)$

(c) Let $C$ be a circle whose radius is $r$ and whose diameter is $d$.
   i. The circumference of $C$ as a function of $r$ is $2\pi r$
   ii. The circumference of $C$ as a function of $d$ is $\pi d$
   iii. The area of $C$ as a function of $r$ is $\pi r^2$
   iv. The area of $C$ as a function of $d$ is $\frac{\pi d^2}{4}$

10. (a) $f(n)$ (* $n \geq 1$ is an integer *)
    
    $c = 0$
    
    for $i = 1$ to $n$ do
    
    for $j = 1$ to $n$ do
    
    $c := c + 1$
    
    $\implies c = n^2$

(b) $f(n)$ (* $n \geq 1$ is an integer *)
    
    $c = 0$
    
    for $i = 1$ to $n$ do
    
    for $j = i$ to $n$ do
    
    $c := c + 1$
    
    $\implies c = n + (n - 1) + (n - 2) + \cdots + 1 = \frac{n(n+1)}{2}$

(c) $f(n)$ (* $n \geq 1$ is an integer *)
    
    $c = 1$
    
    for $i = 1$ to $n$ do
    
    $c := c \times 2$
    
    $\implies c = 2^n$

(d) $f(n)$ (* $n \geq 1$ is a power of 2 integer *)
    
    $c = 0$
    
    while $n > 1$ do
    
    $n := n/2$
    
    $c := c + 1$
    
    $\implies c = \log_2(n)$
11. (a) \[ 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} \]
(b) \[ 1 + 2 + 4 + 8 + \cdots + 2^n = 2^{n+1} - 1 \]
(c) \[ \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n} = 1 - \frac{1}{2^n} \]

12. The following functions are ordered (left to right) by their growth from the slowest to the fastest when \( n \) tends to infinity:

\[
1 < \log \log(n) < \log(n) < n < n^2 < 2^n < n! < n^n
\]