Neural Networks

Introduction

CIS 32

Functionalia

Office Hours (Last Change!) - Location Moved to 0317 N (Bridges Room)

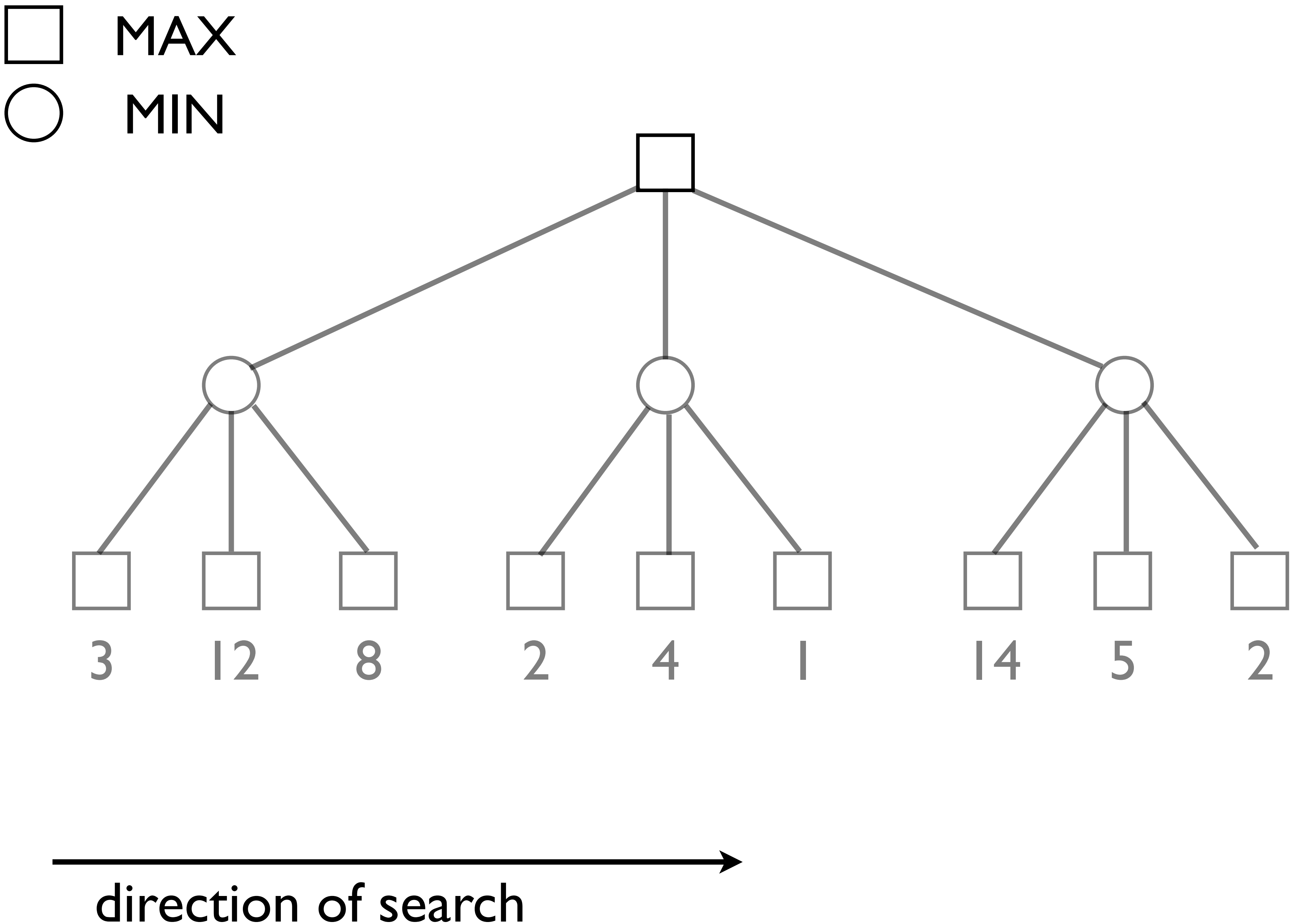
Today:

Alpha-Beta Example

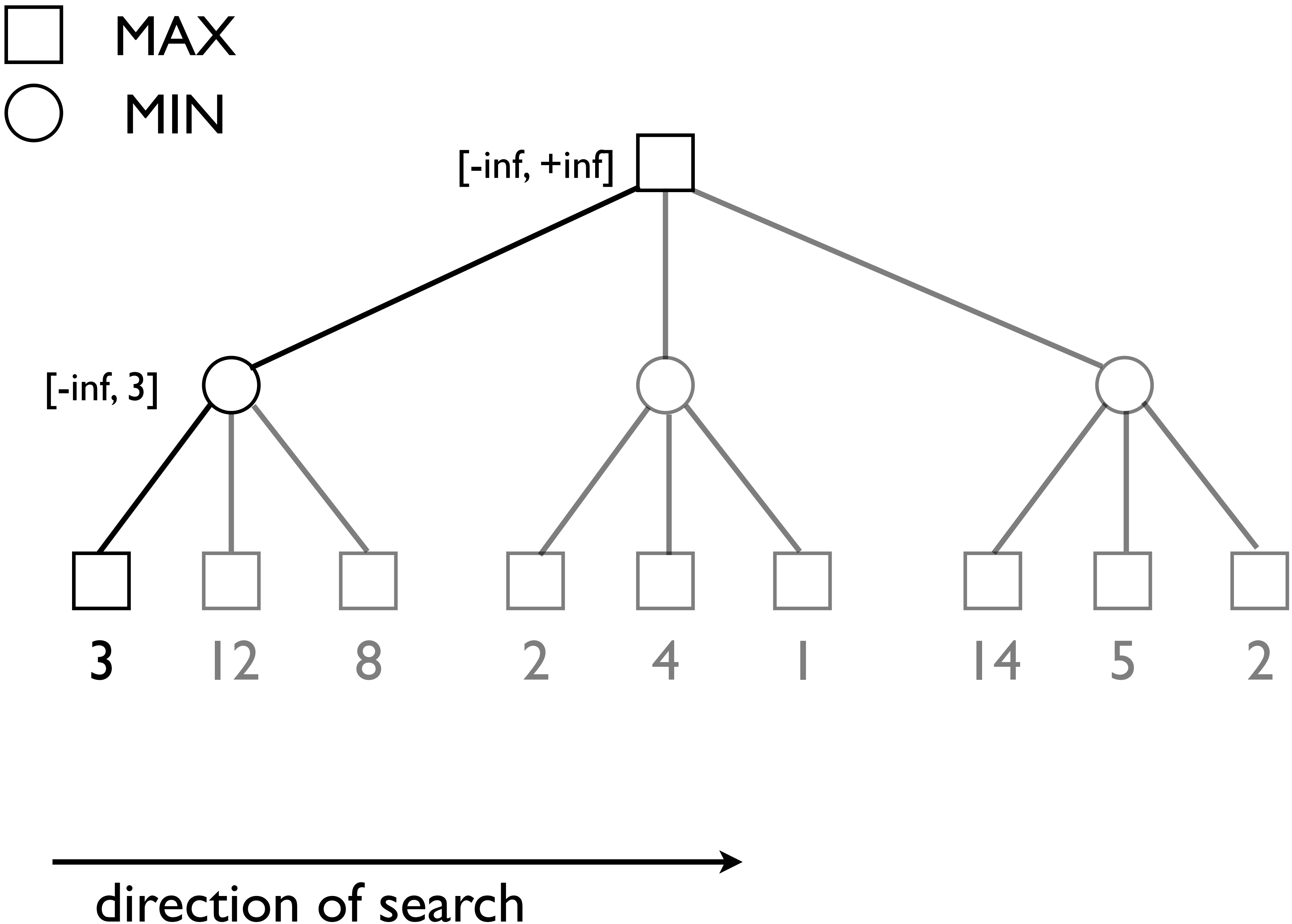
Neural Networks

Learning with T-R Agent (from before)

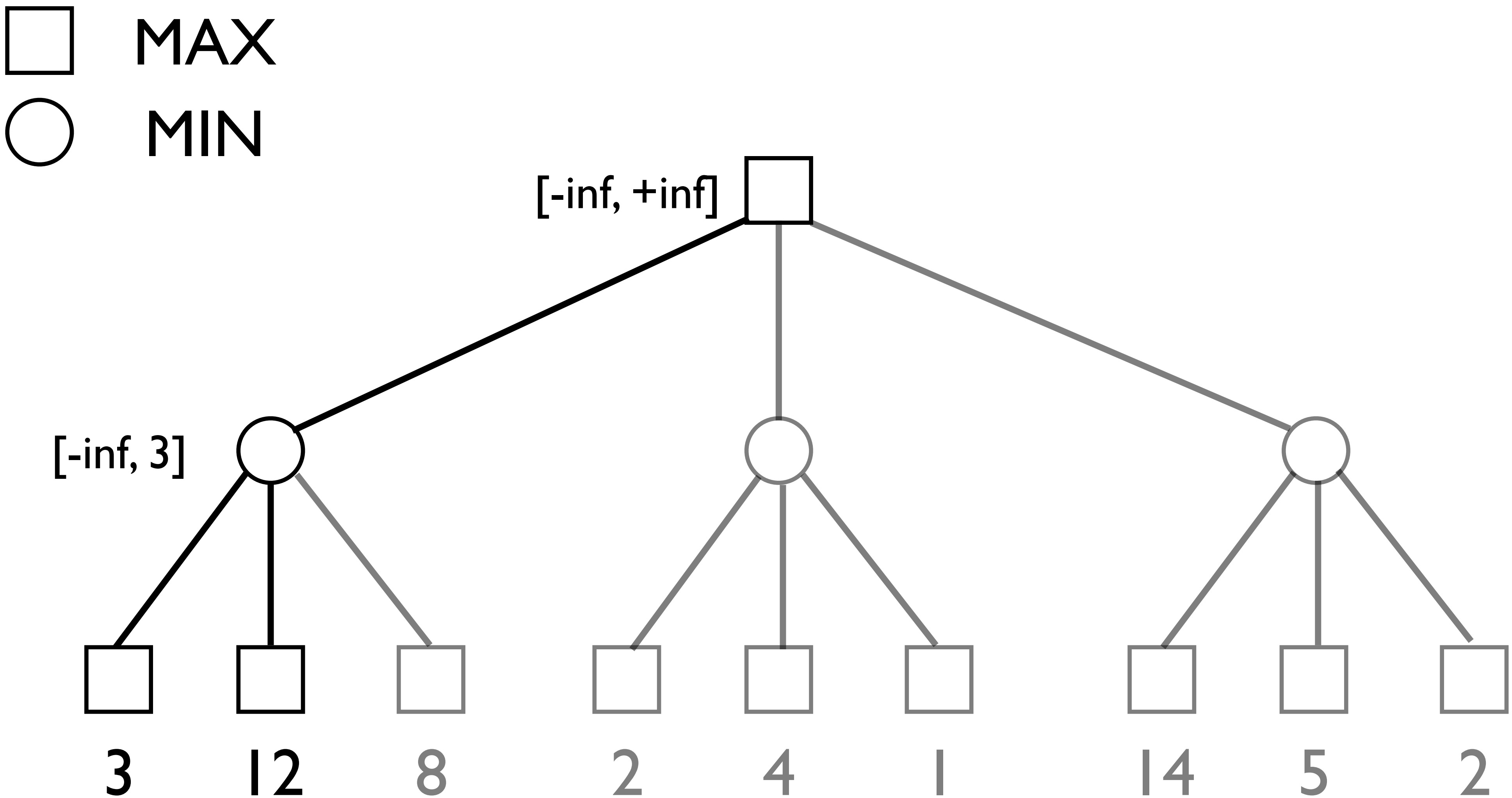
Alpha-Beta Example



Alpha-Beta Example

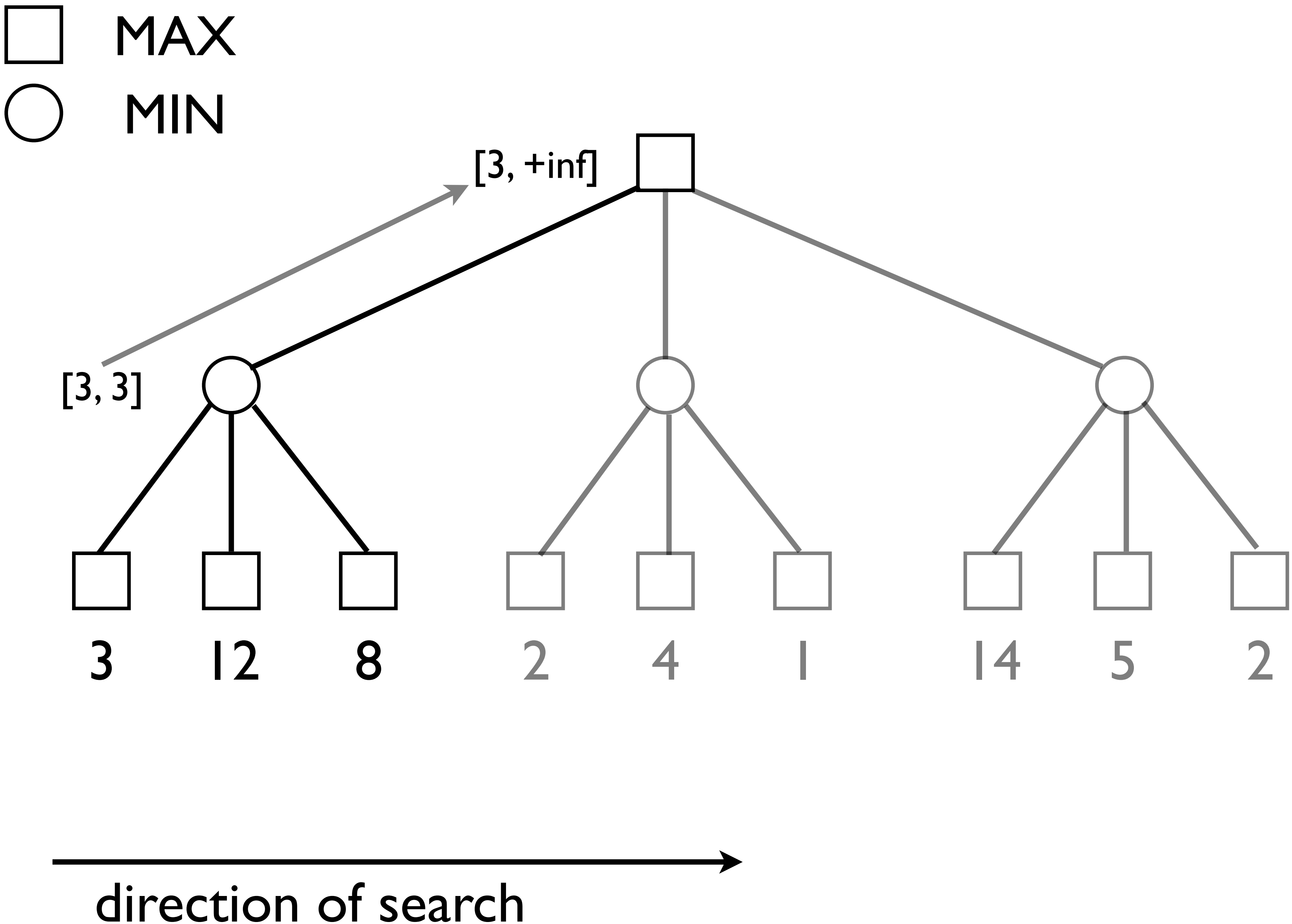


Alpha-Beta Example

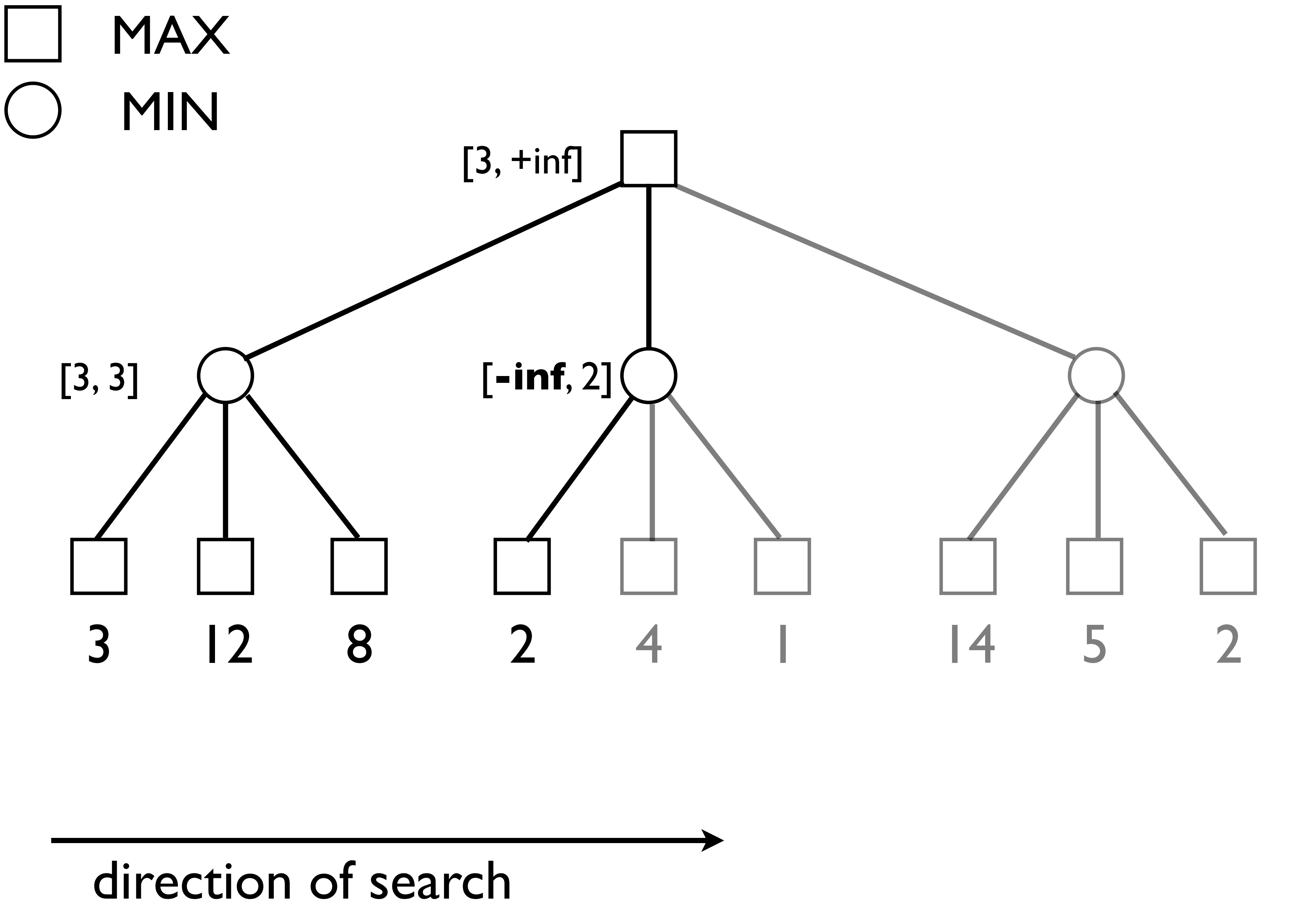


direction of search

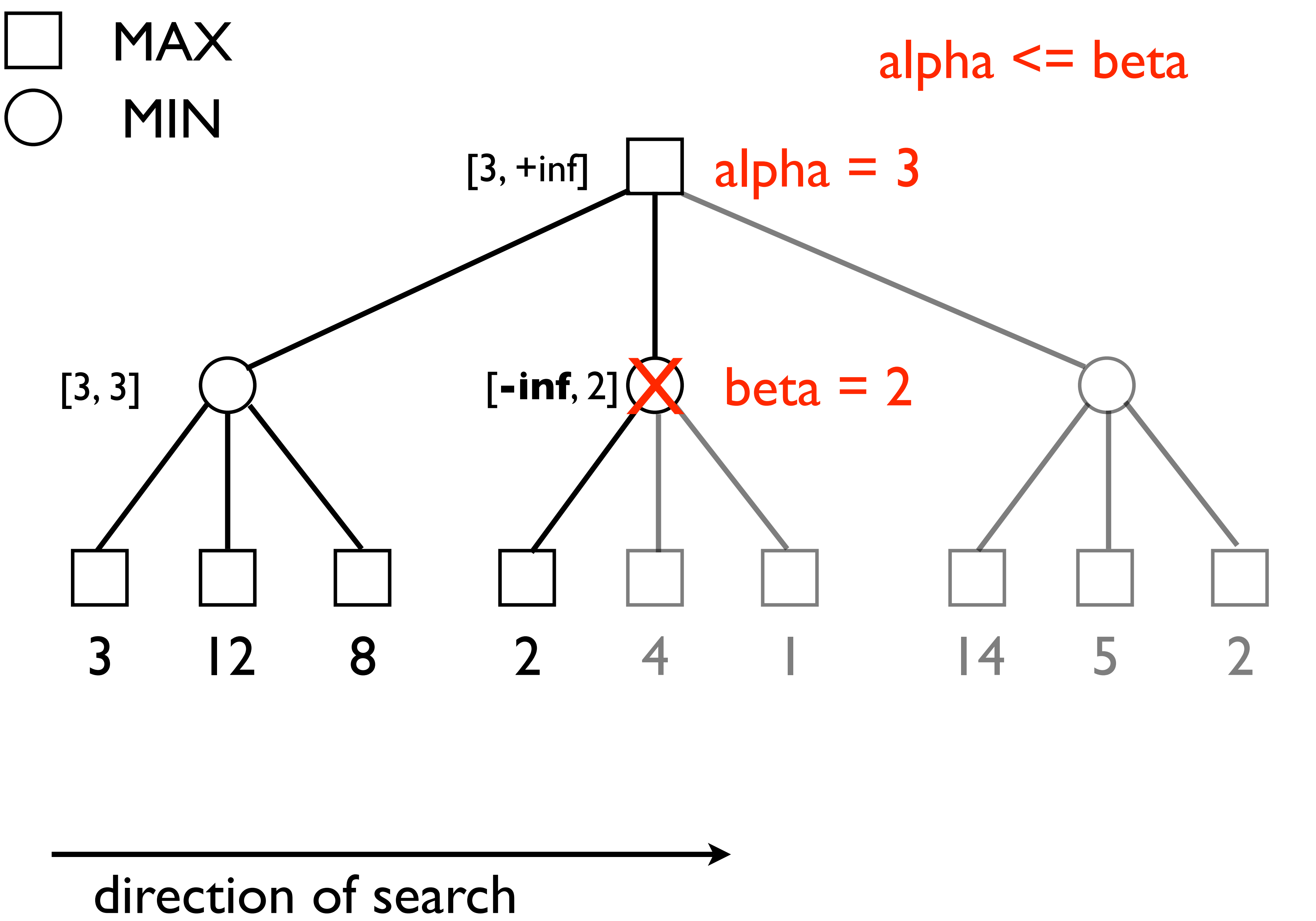
Alpha-Beta Example



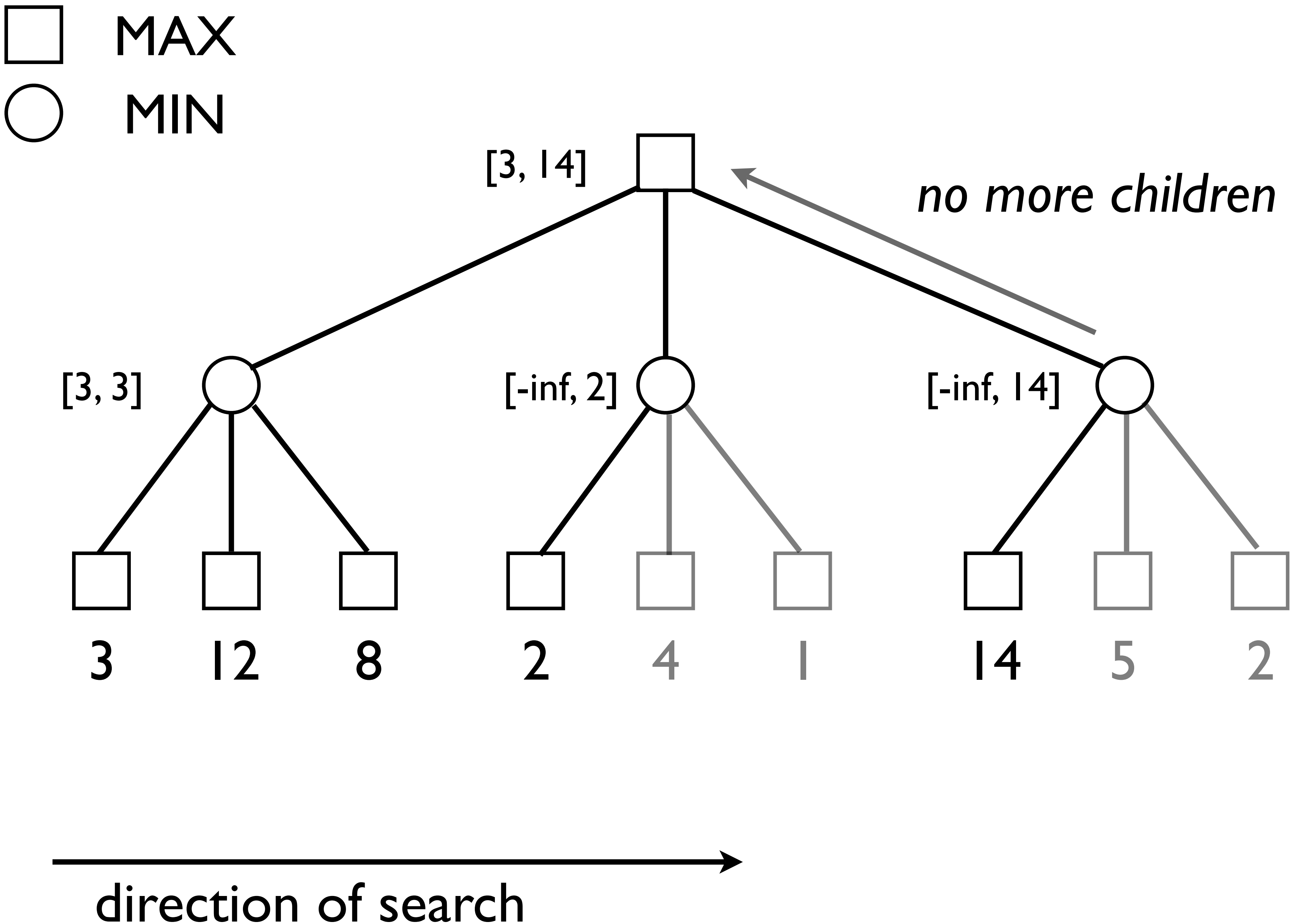
Alpha-Beta Example



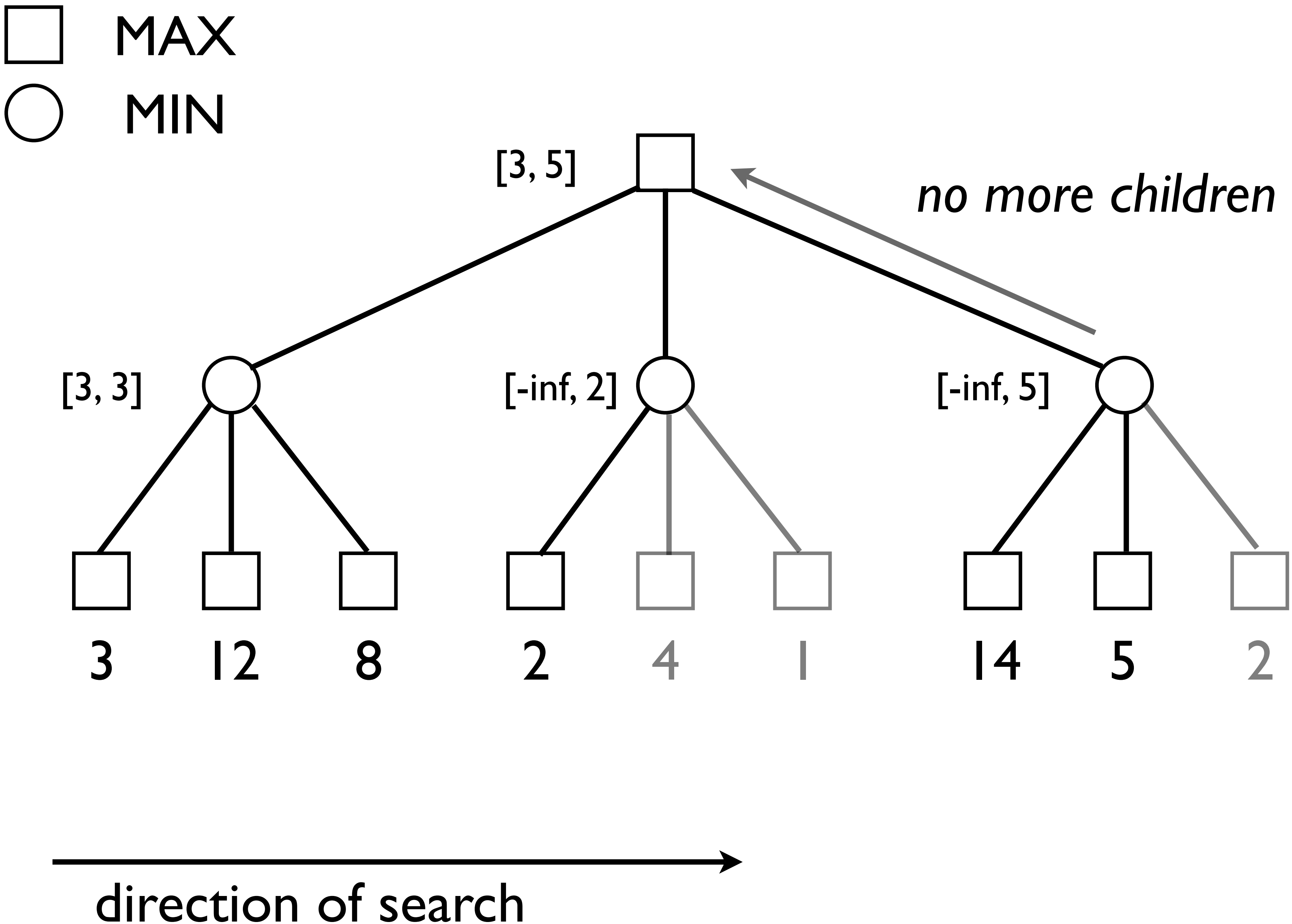
Alpha-Beta Example



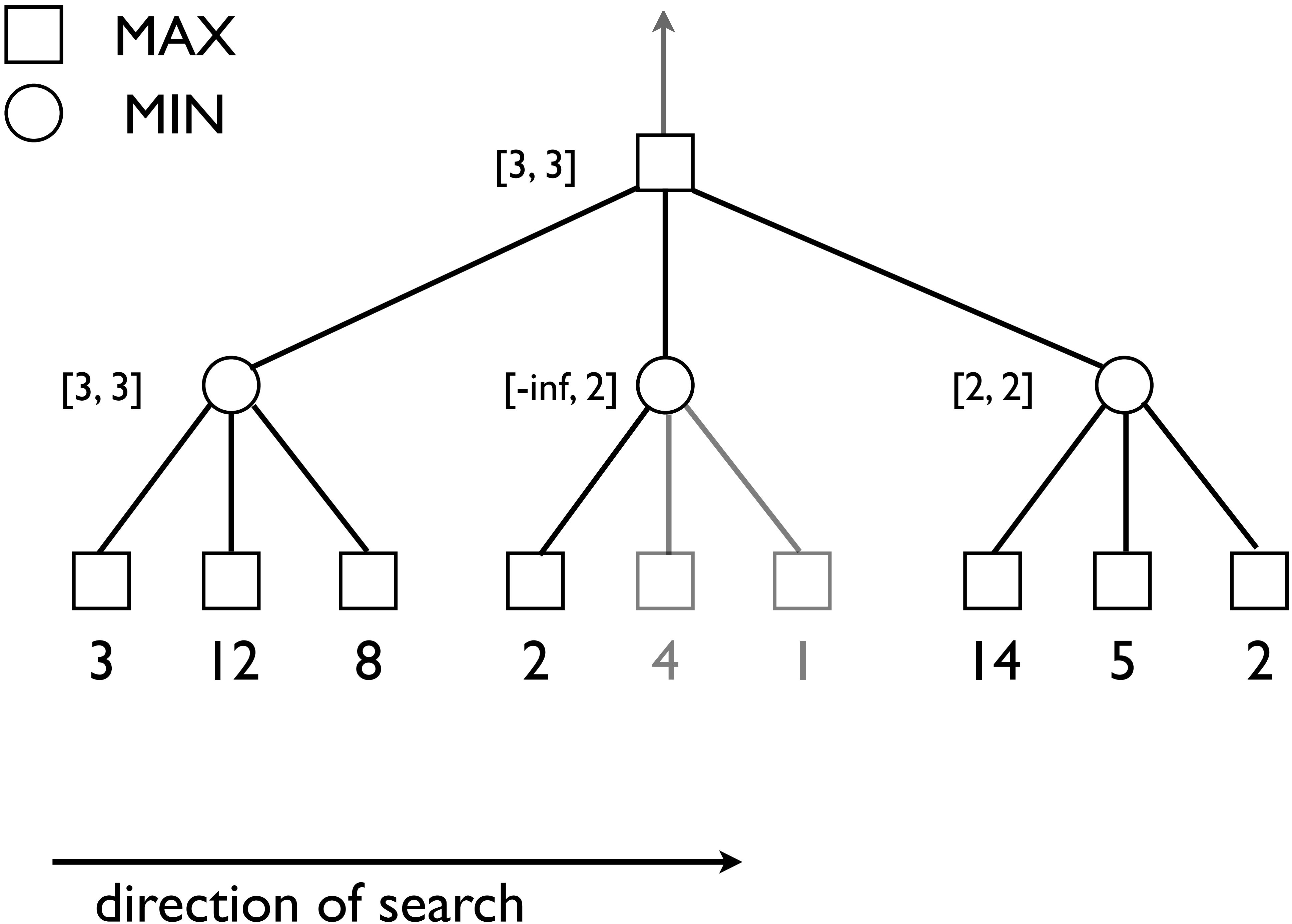
Alpha-Beta Example



Alpha-Beta Example

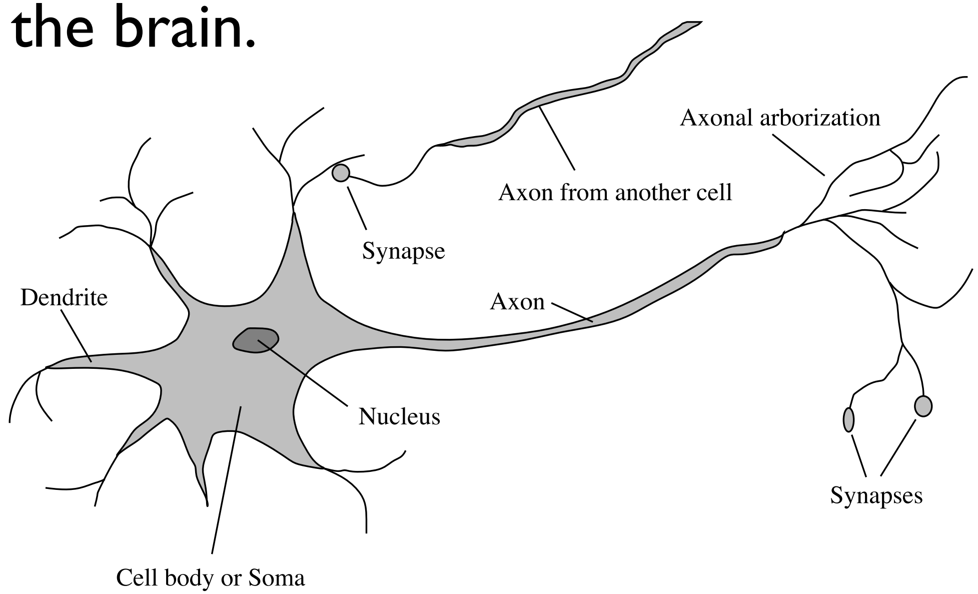


Alpha-Beta Example



Neural Networks: Introduction

• Now we will look at neural networks, so called because they mimic the structure of the brain.



• However, they don't have to be viewed in this way.

• We will start by thinking of them as an implementation of the kind of stimulus-response agents we looked at in the last lecture.

• They also provide us with our first taste of learning.

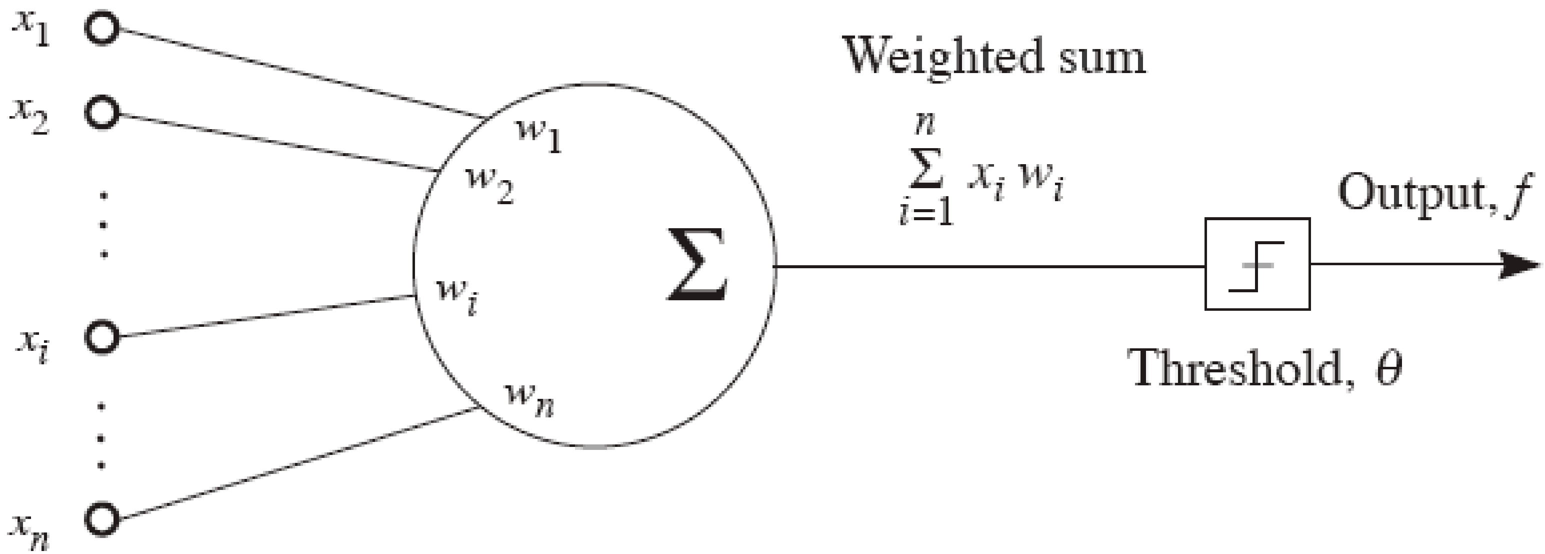
• The learning angle means we don't have to figure out the model parameters for ourselves.

Networks for Stimulus-Response

- Production systems can be easily implemented as computer programs.
- They may also be implemented directly as electronic circuits, as combinations of AND, OR, and NOT gates.

  (Or as simulations of electronic circuits.)

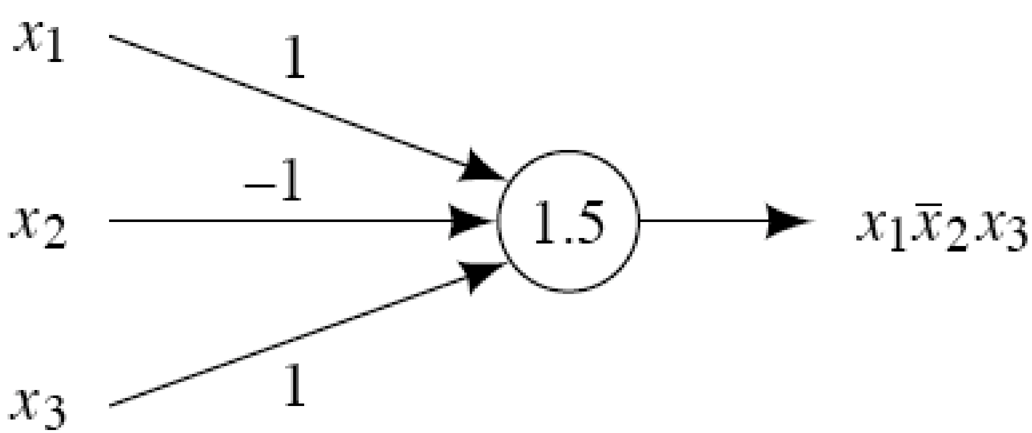
- One useful kind of circuit is built of elements whose output is a nonlinear discrete function of a weighted combinations of its inputs.
- One kind of such unit is a **threshold logic unit** (TLU).
- This computes a weighted sum of its inputs, compares this to a threshold, and outputs 1 if the threshold is exceeded, 0 otherwise.



$$
\begin{aligned}
f &= 1 \text{ if } \sum_{i=1}^{n} x_i w_i \geq \theta \\
&= 0 \text{ otherwise}
\end{aligned}
$$

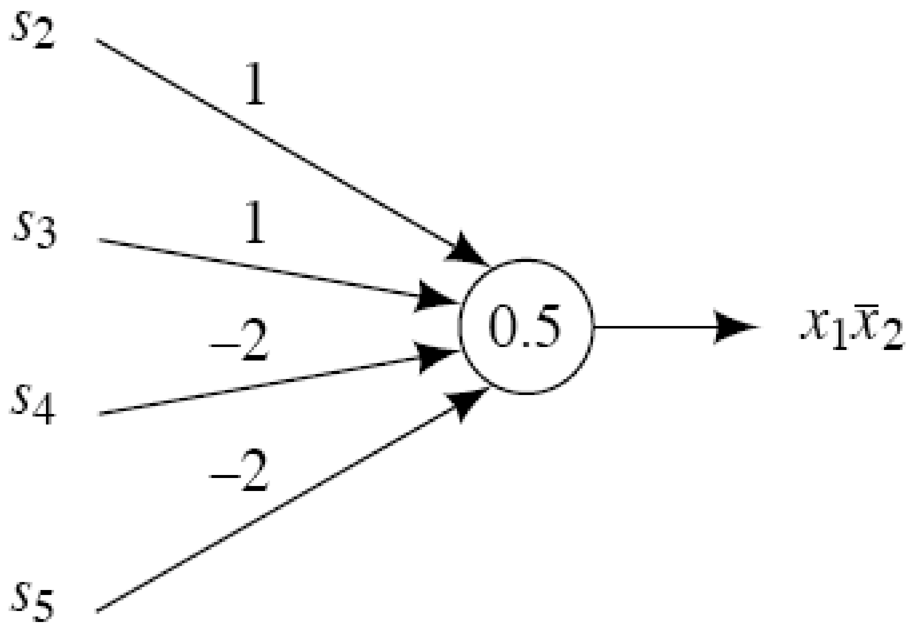
• The Boolean functions that can be computed using a TLU are called linearly separable functions.

• We can use TLUs to implement some Boolean functions, for instance a simple conjunction:



but we can't implement an exclusive-OR this way.
(Because an XOR is not *linearly separable*. We will see...)

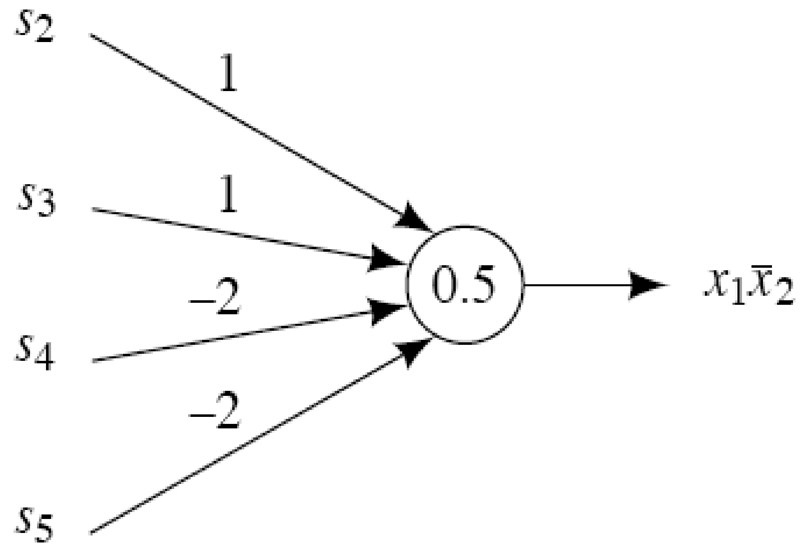
Boundary Following Agent (from before)



$$
\begin{aligned}
x_1\overline{x_2} &= (s_2 + s_3)\overline{(s_4 + s_5)} \\
&= (s_2 + s_3)\overline{s_4 s_5}
\end{aligned}
$$

- We can implement the kind of function used for boundary following.

s2	s3	s4	s5	Activation	$x_1\overline{x_2}$
0	0	0	0	0(1)+0(1)+0(-2)+0(-2) = 0	0
0	0	0	1	0(1)+0(1)+0(-2)+1(-2) = -2	0
0	0	1	0	0(1)+0(1)+1(-2)+0(-2) = -2	0
0	0	1	1	0(1)+0(1)+1(-2)+1(-2) = -4	0
0	1	0	0	0(1)+1(1)+0(-2)+0(-2) = 1	1
0	1	0	1	0(1)+1(1)+0(-2)+1(-2) = -1	0
0	1	1	0	0(1)+1(1)+1(-2)+0(-2) = -1	0
0	1	1	1	0(1)+1(1)+1(-2)+1(-2) = -3	0
1	0	0	0	1(1)+0(1)+0(-2)+0(-2) = 1	1
1	0	0	1	1(1)+0(1)+0(-2)+1(-2) = -1	0
1	0	1	0	1(1)+0(1)+1(-2)+0(-2) = -1	0
1	0	1	1	1(1)+0(1)+1(-2)+1(-2) = -3	0
1	1	0	0	1(1)+1(1)+0(-2)+0(-2) = 2	1
1	1	0	1	1(1)+1(1)+0(-2)+1(-2) = 0	0
1	1	1	0	1(1)+1(1)+1(-2)+0(-2) = 0	0
1	1	1	1	1(1)+1(1)+1(-2)+1(-2) = -2	0

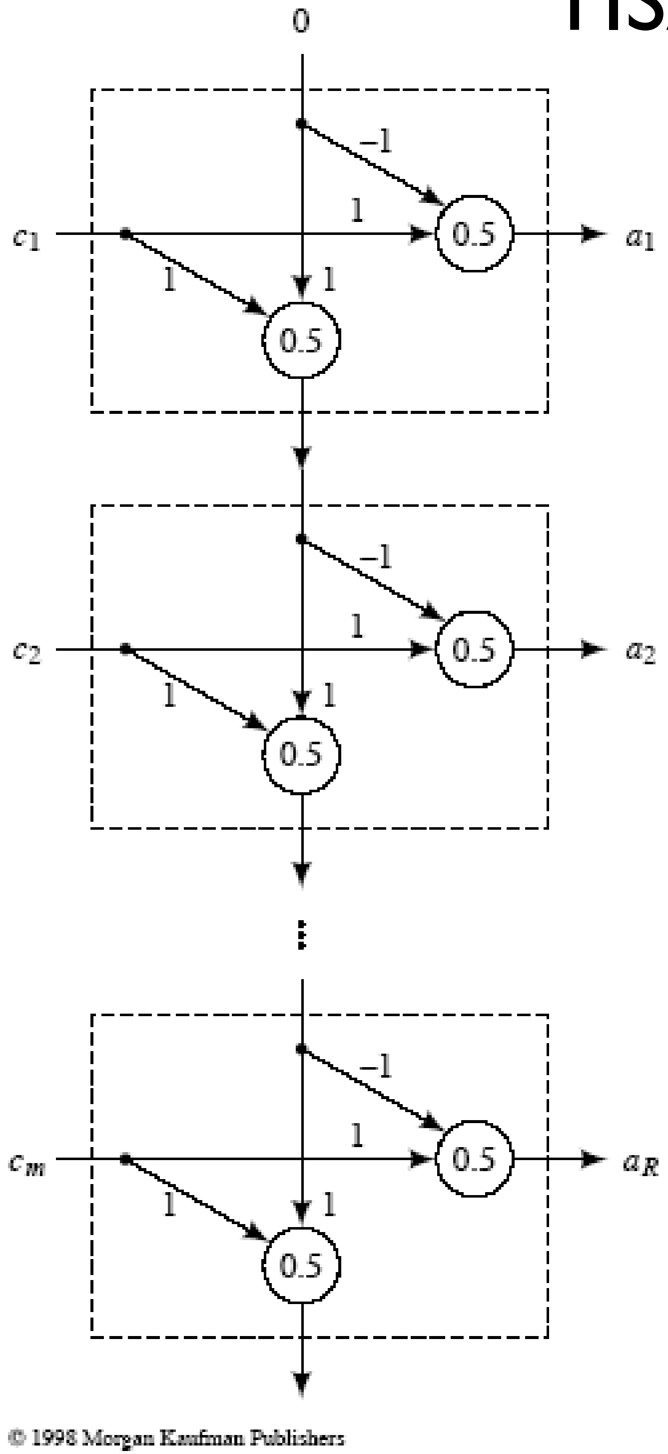


$$
\begin{aligned}
x_1\overline{x_2} &= (s_2+s_3)\overline{(s_4+s_5)} \\
&= (s_2+s_3)\overline{s_4}\overline{s_5}
\end{aligned}
$$

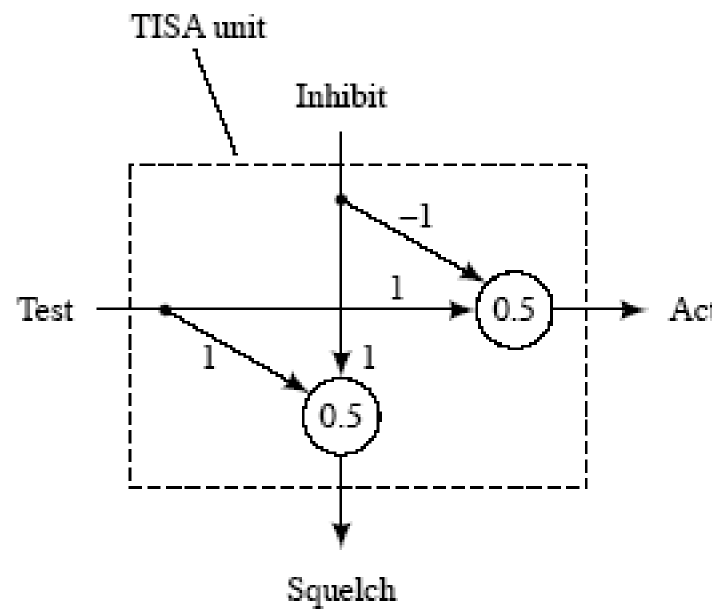
Simple TLU

- When we have a simple problem, it is possible that a single TLU can compute the right action.
- For this to happen we need there to be only two possible actions.
- For more complex problems (categories), we need a network of TLUs.
- These are often called *neural networks* because they have some similarity to the networks of neurons from which the brain is constructed.
- We can use such a network to implement a T-R program.

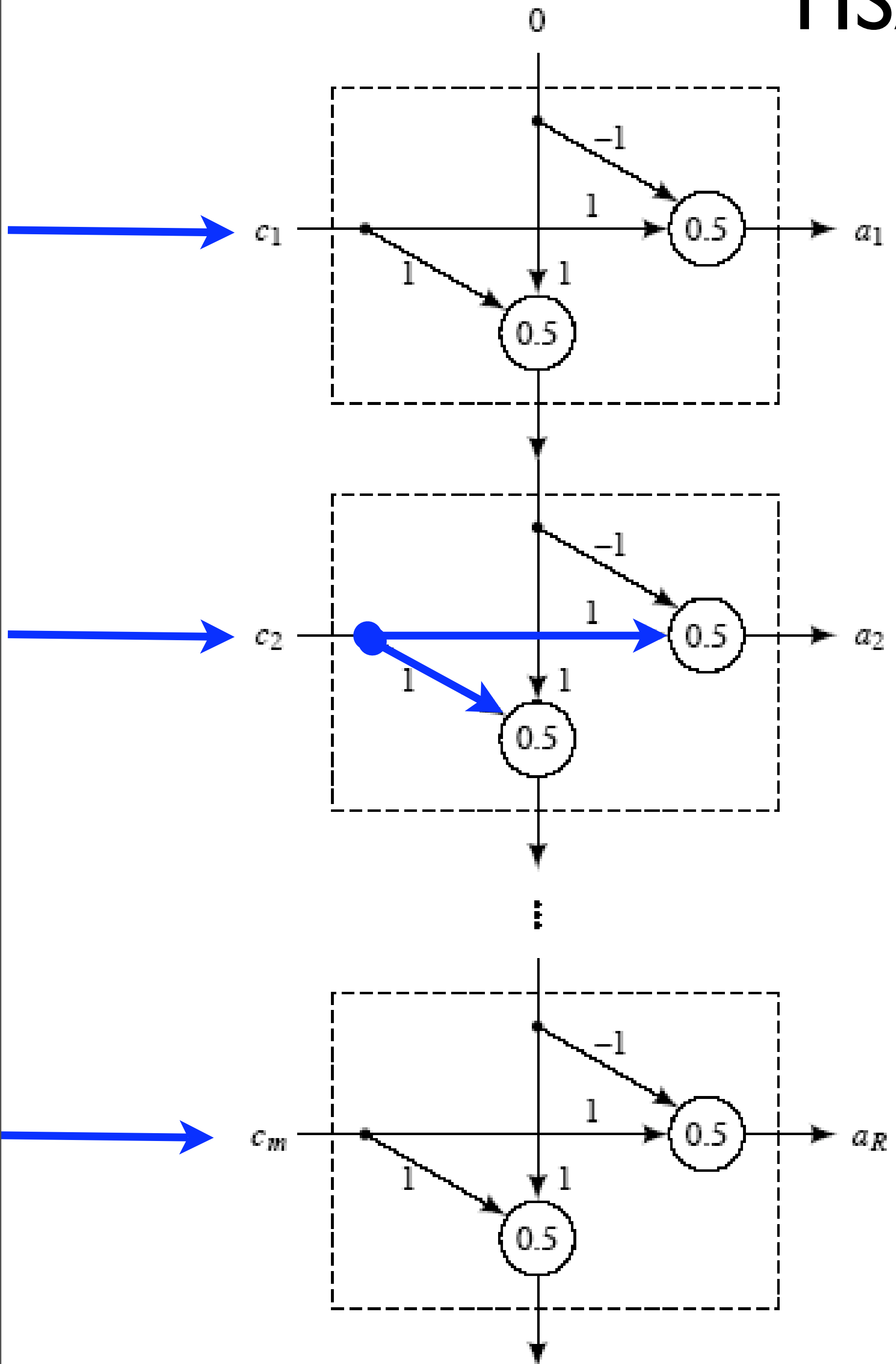
TISA Network



This network implements a set of production rules.



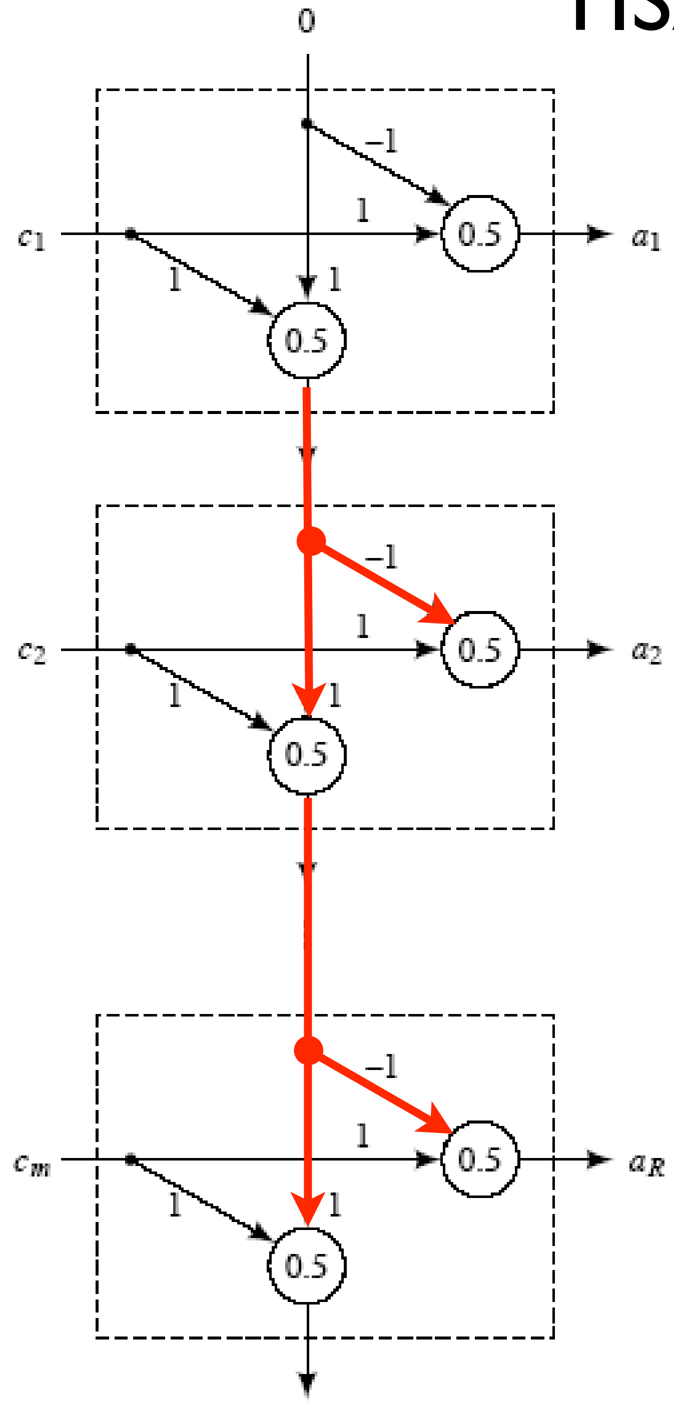
TISA Network



Inputs The input to each unit on the left is the 1 or 0 of the condition.

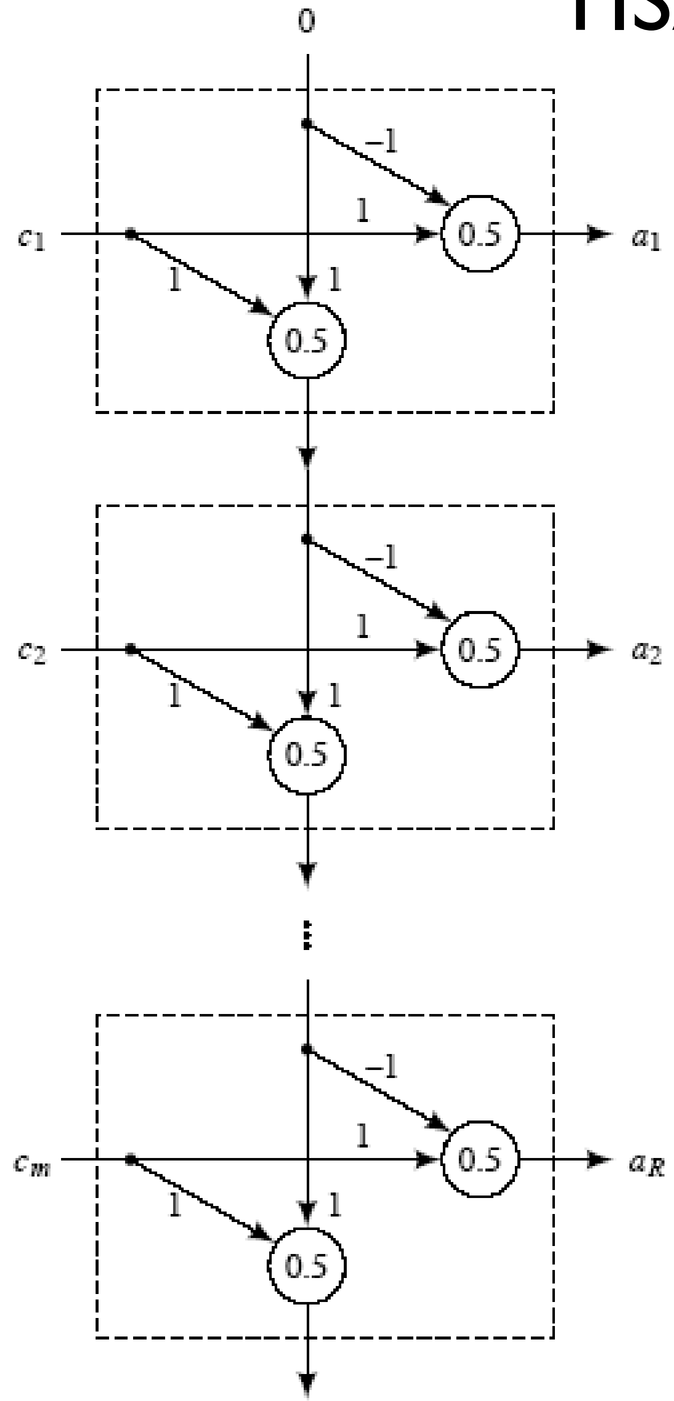
(This might be computed from the s_i by another circuit.)

TISA Network



Inputs The input to each unit from the top is a 1 or a 0 Inhibit input from another Unit.

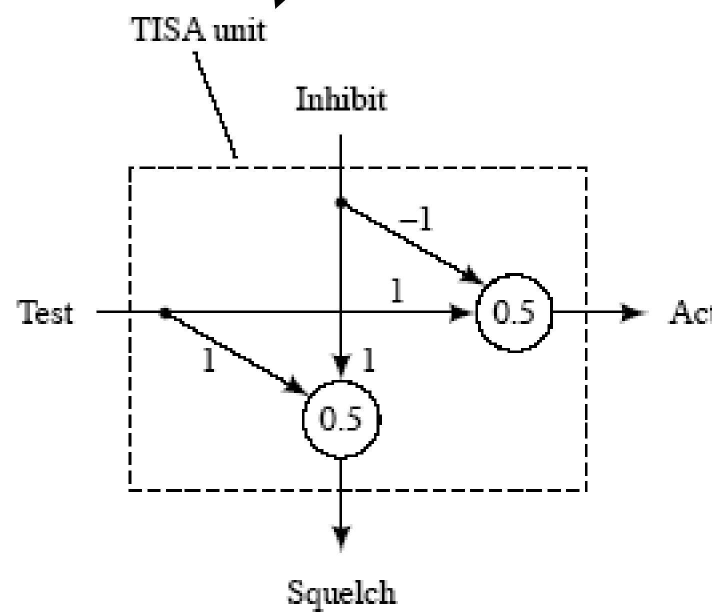
TISA Network



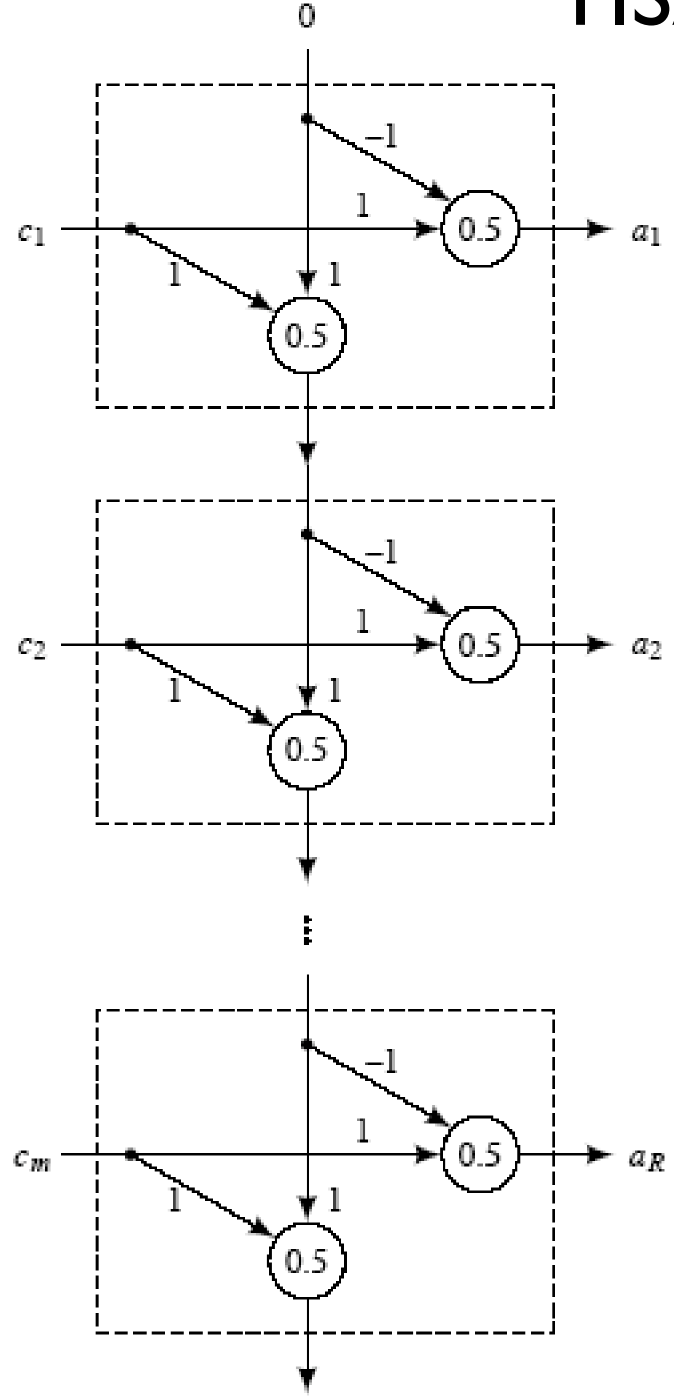
Each rule is a:

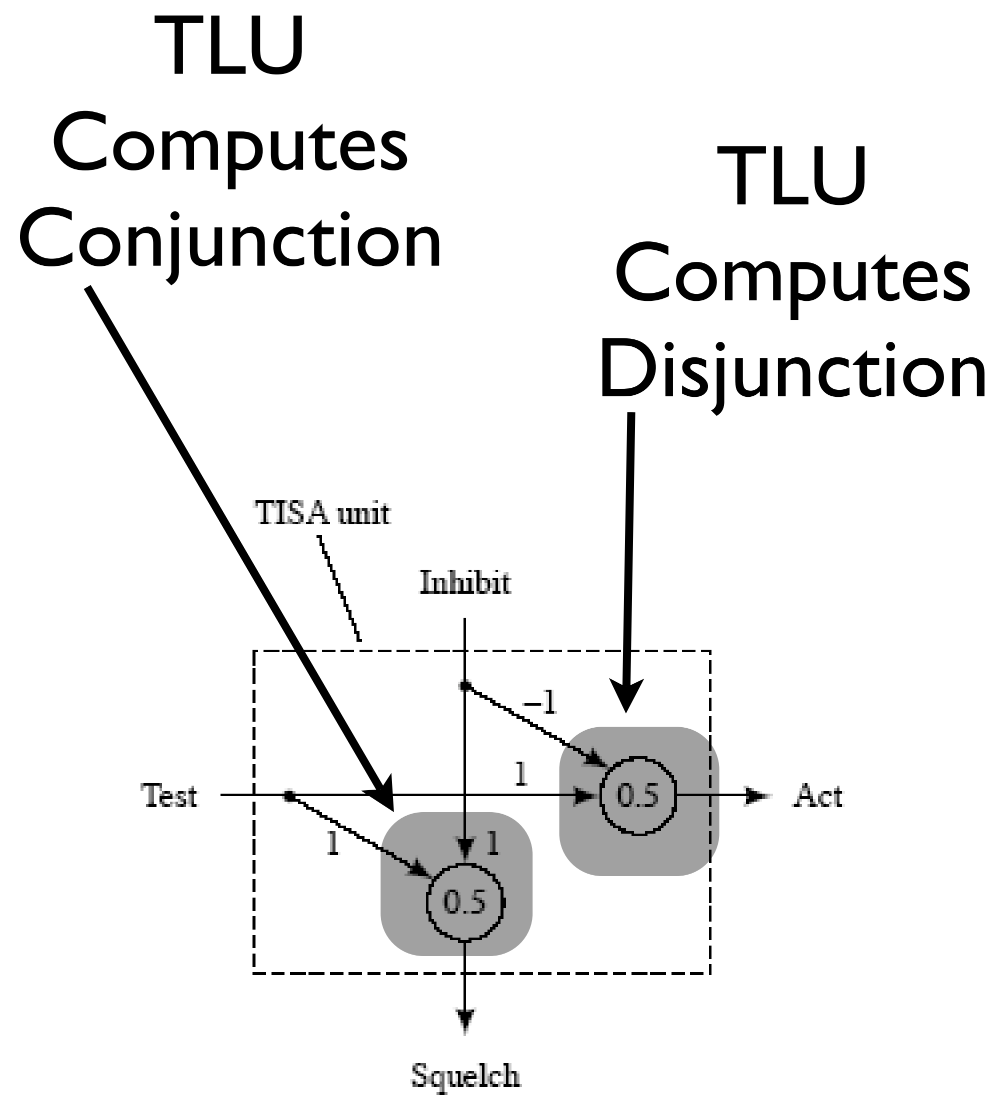
Test,
Inhibit,
Squelch,
Act

(TISA) circuit



TISA Network





TISA Behavior

- Inhibit is 0 when no rules above have a true condition.
- Test is 1 if the condition is true.
- If Test is 1 and Inhibit is 0, Act is 1.
- If either Test is 1 or Inhibit is 1 then Squelch is 1.
- If Squelch is 1 then every TISA below is Inhibited.

Learning in neural networks

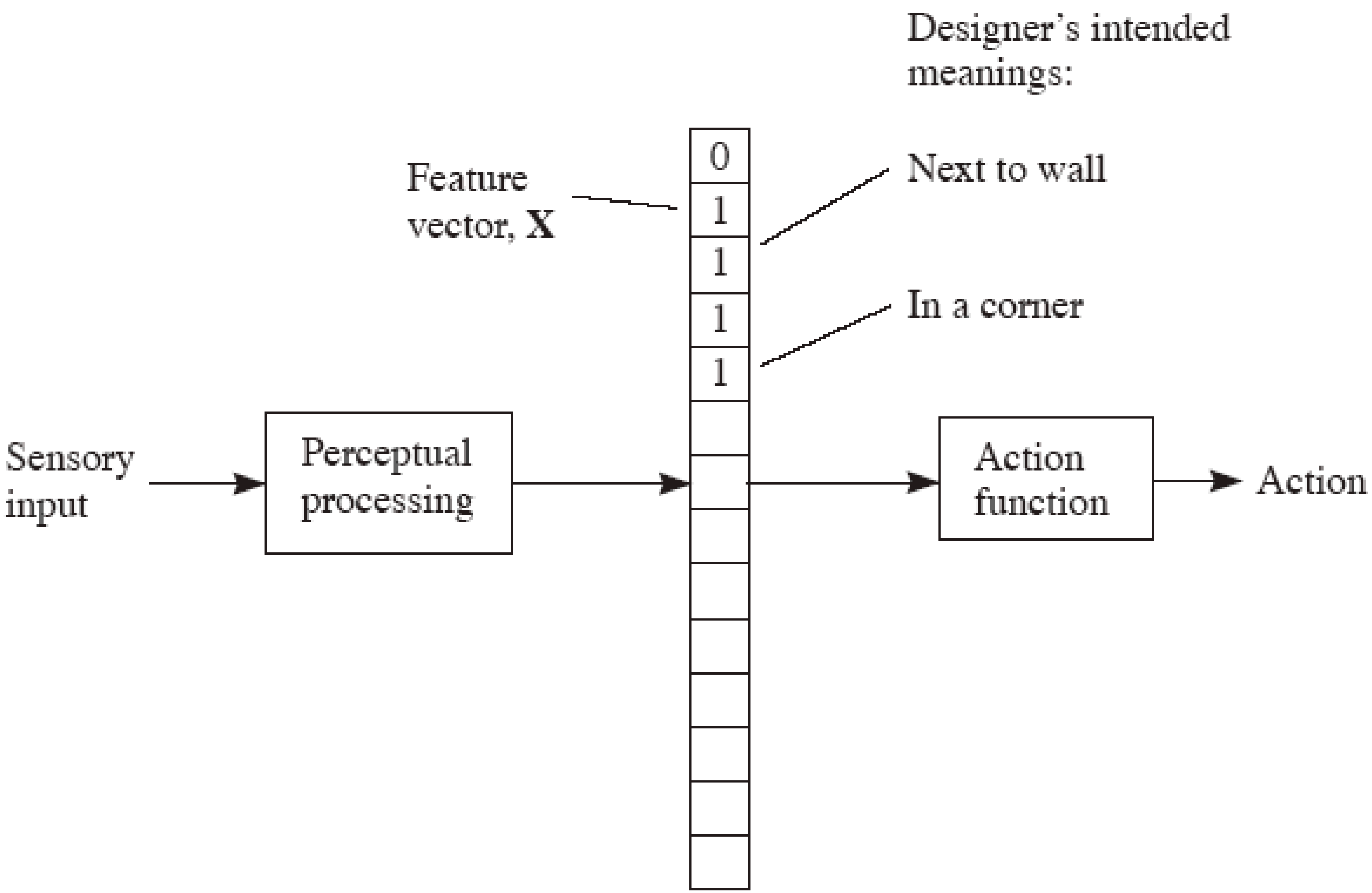
- So far we have assumed that the mapping between stimulus and response was programmed by the agent designer.
- That is not always convenient or possible.
- When it isn't, then it is possible to *learn* the right mapping.
- We will look at the case of learning the mapping for a single TLU.

Learning Process

- In brief, the learning procedure is as follows.
- We start with some set of weights:
  - random;
  - uniform
- **Use a Training Set:** Run a set of inputs, and look at the outputs:
  - If they don't match, we alter the weights.
  - We keep learning until the weights are right.

Back to the T-R Agent from Before

Feature Vector ***X***, maps to a particular action, ***a***:



Supervised Learning

- Now consider that we have a set of these Θ.
- Every element of Θ is an ***X*** (feature vector) with a corresponding ***a*** (action).
- This is a training set, and the set ***A*** of all ***a*** are called the *classes* or *labels*.
- The learning problem here is to find a way of describing the mapping from each member of Θ to the appropriate member of ***A***.
- We want to find a function *f(**X**)* which is "*acceptable*".
- That is it produces an action which agrees with the examples for as many members of the training set as possible.
- Because we have a set of examples to learn from, we call this *supervised learning*.

Learning in a single TLU

- We train a TLU by adjusting the input weights.
- We assume that the vector X is numerical so that a weighted sum makes sense.
- The set of weights $w_1, w_2, \ldots, w_n$ is denoted by vector W
- The threshold is written as θ, so:

– Output is 1 if

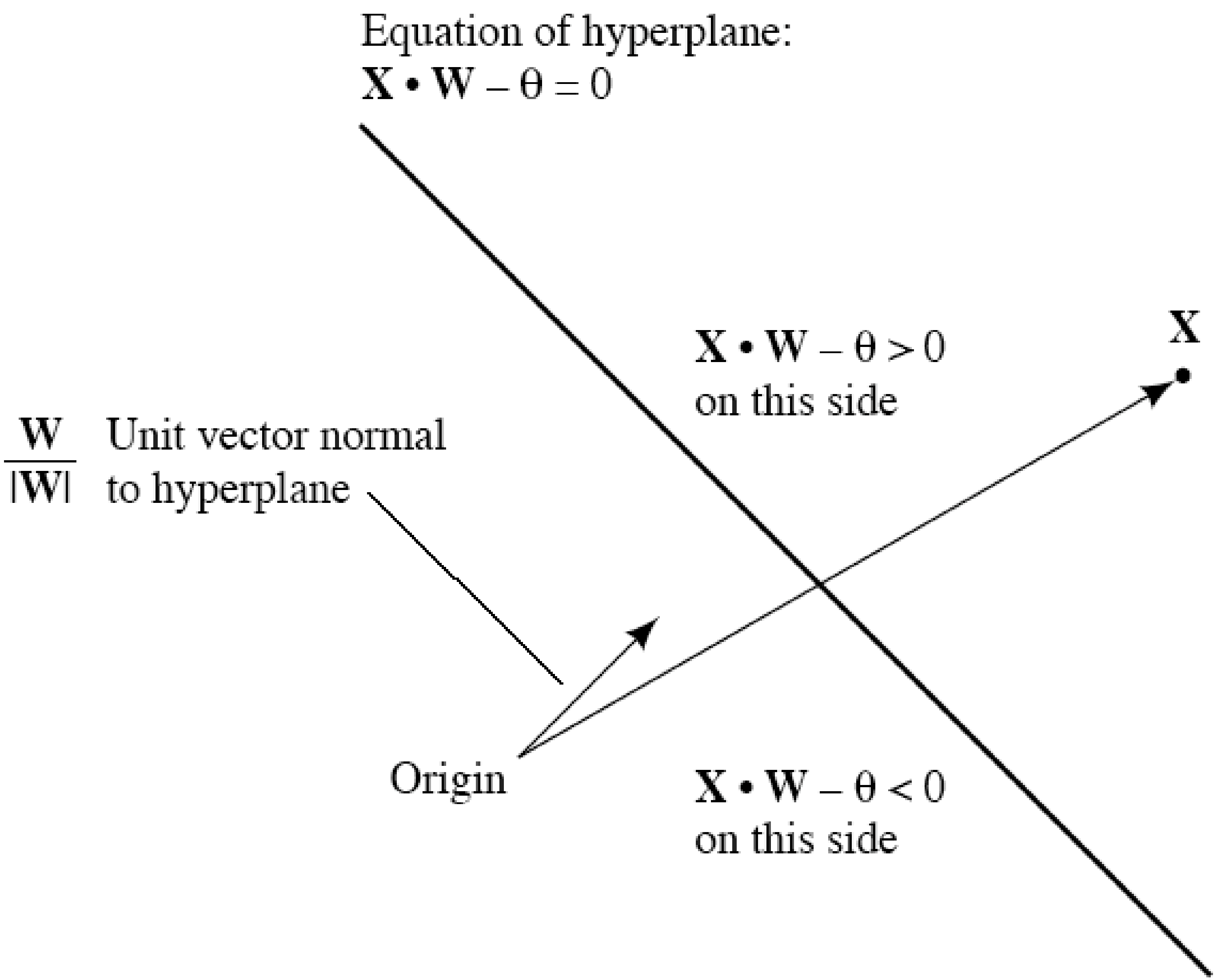
$$s = X \cdot W > \theta$$

– Output is 0 otherwise

- **Dot Product**: $X \cdot W$ is just a way of writing $x_1w_1 + x_2w_2 + \ldots + x_nw_n$

Hyperplane

- A TLU divides the space of feature vectors Θ



• In two dimensions, the TLU defines a boundary (a line) between two parts of a plane (as in the previous picture).

• In three dimensions, the TLU defines a plane which separates two parts of the space.

• In higher-dimension spaces the boundary defined by the TLU is a *hyperplane*.

• Whatever it is, it separates:

$$X \cdot W - \theta > 0$$

from

$$X \cdot W - \theta < 0$$

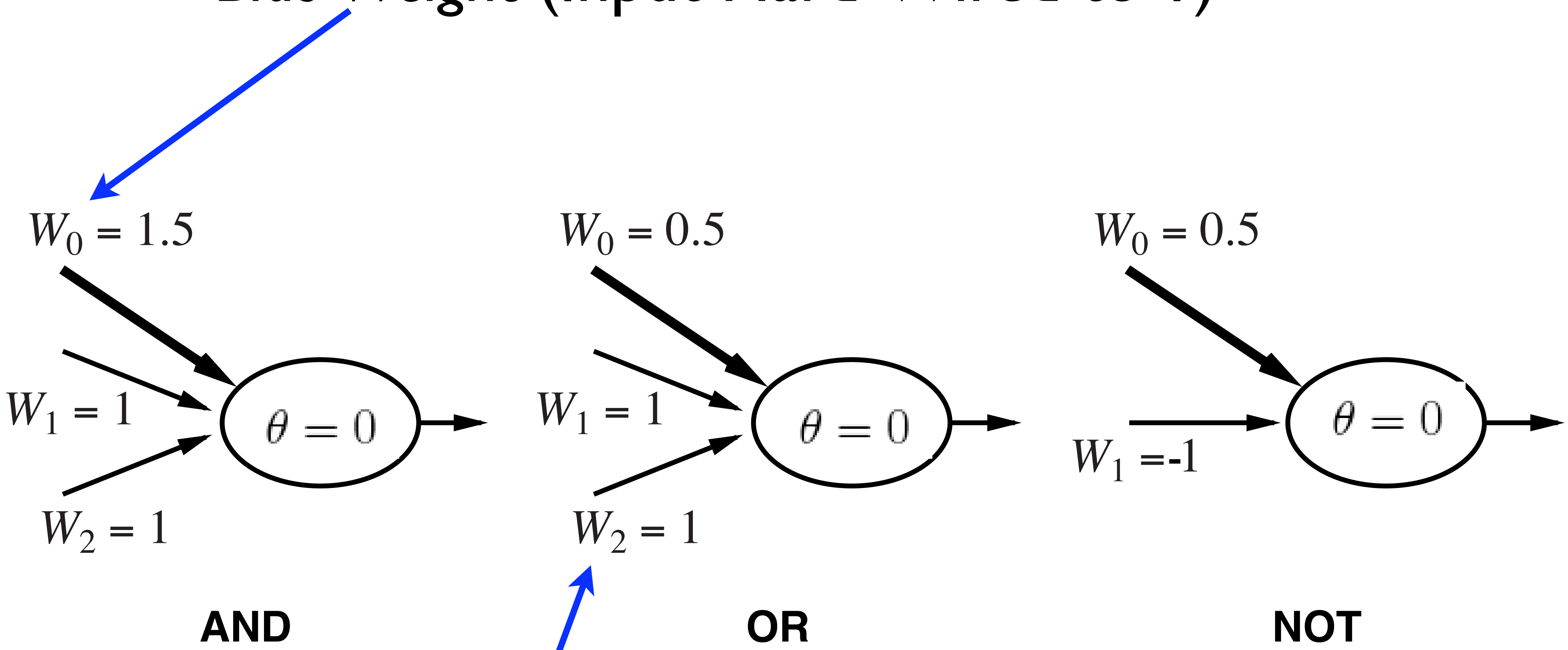
- Changing θ moves the boundary relative to the origin.
- Changing W alters the orientation of the boundary.
- By convention we will assume that:

$$\theta = 0$$

- Simplifies the maths :-)
- Arbitrary thresholds can be obtained by adding in an extra weight $n+1$ which is $-\theta$
- The $n+1$th element of the input vector is always 1.
- So, we don't restrict ourselves by making this assumption.

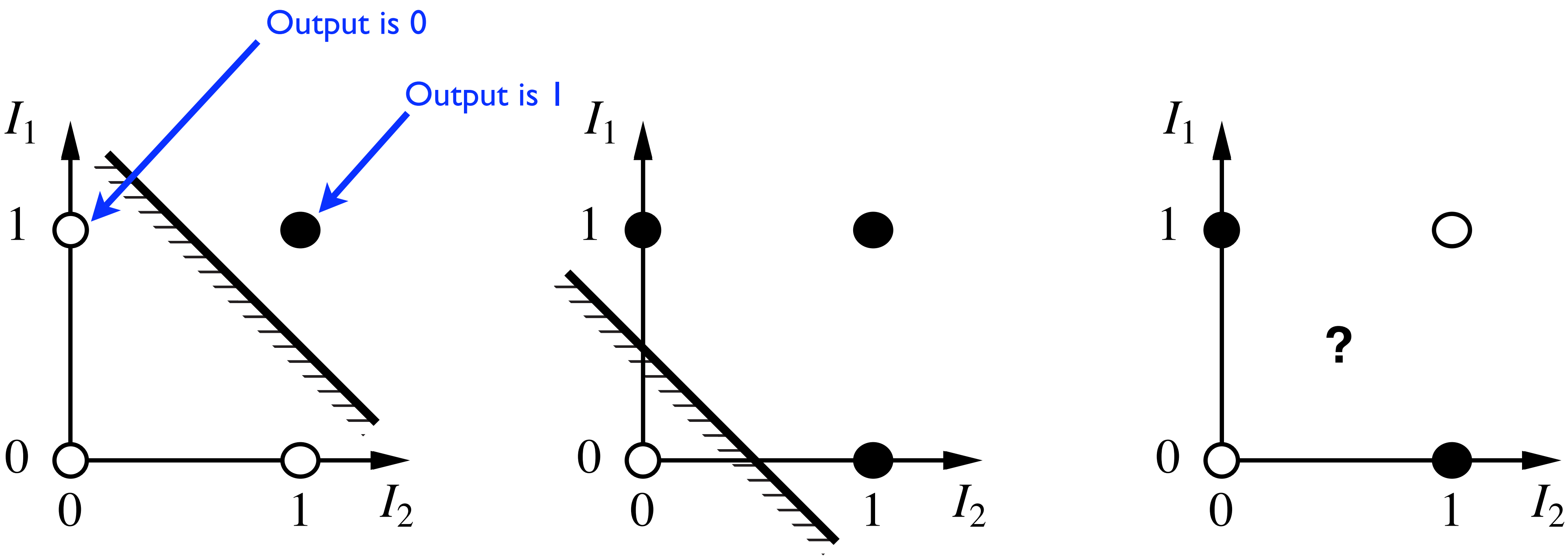
Demonstrated in Logic Functions

Bias-Weight (Input Hard-Wired to 1)



Logic Function Inputs

Linear Separability



(a) I_1 **and** I_2 (b) I_1 **or** I_2 (c) I_1 **xor** I_2

Boundary is clear for **AND** and **OR**, but not for **XOR**.

Summary: Neural Networks

- So, we introduced neural networks.
- We first considered them as an implementation of stimulus-response agents.
- In this incarnation we adjust the weights by hand.
- We also started thinking about how to learn the weights automatically.