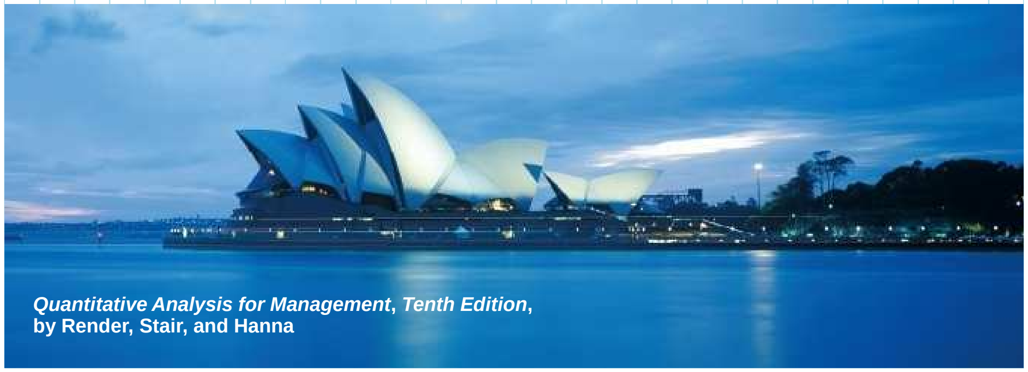
Chapter 3

Decision Analysis



Introduction

- The success of a personal career or business operation is largely dependent on the decisions the person or the business makes
- Decision theory is an analytic and systematic approach to the study of decision making – use mathematical models
- A good decision is one that is based on logic, considers all available data and possible alternatives, and uses the quantitative approach (mathematical models) described here

Introduction

- Occasionally, a good decision may result in an unexpected or unfavorable outcome.
- A bad decision is one that is not based on logic, does not use all available. information, does not consider all alternatives, and does not employ appropriate quantitative techniques.
- A bad decision can sometimes result in a favorable outcome out of luck.
- In a long run, using decision theory will result in successful outcomes.

The Six Steps in Decision Making

1. Clearly define the problem at hand
2. List the possible alternatives
3. Identify the possible outcomes or states of nature
4. List the payoff or profit of each combination of alternatives and outcomes
5. Select one of the mathematical decision theory models
6. Apply the model and make your decision

Thompson Lumber Company

Step 1 – Define the problem

- Identify whether to expand product line by manufacturing and marketing a new product, backyard storage sheds

Step 2 – List ALL possible alternatives

- Construct a large new plant
- A small plant
- No plant at all

Step 3 – Identify ALL possible outcomes (states of nature)

- The market could be favorable or unfavorable
- States of nature = outcomes over which decision makers have little or no control

Thompson Lumber Company

Step 4 – List the payoffs

- Identify *conditional values* (payoffs / profits) for large, small, and no plants for the two possible market conditions (Table 3.1)
- Money may not be the only value to look at, other means of measuring benefit is also acceptable

Step 5 – Select the decision model

- Depends on the environment and the amount of risk and uncertainty involved

Step 6 – Apply the model to the data

- Solution and analysis used to help the decision making

Thompson Lumber Company

	STATE OF NATURE	
ALTERNATIVE	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)
Construct a large plant	200,000	–180,000
Construct a small plant	100,000	–20,000
Do nothing	0	0

Table 3.1: Decision Table with Conditional Values for Thompson Lumber

Decision Table and Decision Tree are two ways for decision analysis.

Types of Decision-Making Environments

Type 1: Decision making under certainty

- Decision maker *knows with certainty* the consequences of every alternative or decision choice – so choose the option that will result in the best outcome (e.g. opening savings account vs. buying Treasury bond)

Type 2: Decision making under uncertainty

- The decision maker *does not know* the consequences (even the probabilities) of the various outcomes (weather condition next year)

Type 3: Decision making under risk

- The decision maker *knows the probabilities* of the various outcomes (e.g. rolling a 4 on a die)

Decision Making Under Uncertainty

Most managers are not fortunate enough to make decisions under certainty. There are several criteria for making decisions under uncertainty:

1. Maximax (optimistic)
2. Maximin (pessimistic)
3. Criterion of realism (Hurwicz)
4. Equally likely (Laplace)
5. Minimax regret

Maximax

Used to find the alternative that maximizes the maximum payoff – optimistic approach

- Locate the maximum payoff for each alternative
- Select the alternative with the maximum number

	STATE OF NATURE		
ALTERNATIVE	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	MAXIMUM IN A ROW ($)
Construct a large plant	200,000	–180,000	200,000 ← Maximax
Construct a small plant	100,000	–20,000	100,000
Do nothing	0	0	0

Table 3.2

Maximin

Used to find the alternative that maximizes the minimum payoff – pessimistic approach

- Locate the minimum payoff for each alternative
- Select the alternative with the maximum number

	STATE OF NATURE		
ALTERNATIVE	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	MINIMUM IN A ROW ($)
Construct a large plant	200,000	–180,000	–180,000
Construct a small plant	100,000	–20,000	–20,000
Do nothing	0	0	0 ← Maximin

Table 3.3

Criterion of Realism (Hurwicz)

Maximax and maximin consider only one extreme payoff

A *weighted average* compromises between optimistic and pessimistic approaches

- Select a coefficient of realism α to measure the degree of optimism
- Coefficient is between 0 and 1: a value of 1 is 100% optimistic and 0 is 100% pessimistic
- Compute the weighted averages for each alternative
- Select the alternative with the highest value

Weighted average = α(maximum in row) + (1 – α)(minimum in row)

Criterion of Realism (Hurwicz)

- For the large plant alternative using $\alpha = 0.8$
  (0.8)(200,000) + (1 – 0.8)(–180,000) = 124,000
- For the small plant alternative using $\alpha = 0.8$
  (0.8)(100,000) + (1 – 0.8)(–20,000) = 76,000

	STATE OF NATURE		
ALTERNATIVE	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	CRITERION OF REALISM ($\alpha = 0.8$)$
Construct a large plant	200,000	–180,000	124,000 ← Realism
Construct a small plant	100,000	–20,000	76,000
Do nothing	0	0	0

Table 3.4

Equally Likely (Laplace)

Hurwicz criterion considers the best and worst payoffs only. Laplace criterion considers all the payoffs for each alternative

- Find the average payoff for each alternative
- Select the alternative with the highest average

	STATE OF NATURE		
ALTERNATIVE	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	ROW AVERAGE ($)
Construct a large plant	200,000	–180,000	10,000
Construct a small plant	100,000	–20,000	40,000 ← Equally likely
Do nothing	0	0	0

Table 3.5

Minimax Regret

Based on ***opportunity loss*** or ***regret***, the difference between the optimal profit and actual payoff for a decision – the amount lost by not picking the best alternative

- Create an opportunity loss table by determining the opportunity loss for not choosing the best alternative
- Opportunity loss is calculated by subtracting each payoff in the column from the best payoff in the column
- Find the maximum opportunity loss for each alternative and pick the alternative with the minimum number

Minimax Regret

	STATE OF NATURE	
ALTERNATIVE	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)
Construct a large plant	200,000	–180,000
Construct a small plant	100,000	–20,000
Do nothing	0	0

- Opportunity Loss Tables

STATE OF NATURE	
FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)
200,000 – 200,000	0 – (–180,000)
200,000 – 100,000	0 – (–20,000)
200,000 – 0	0 – 0

Table 3.6

Minimax Regret

Table 3.7

ALTERNATIVE	STATE OF NATURE: FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)
Construct a large plant	0	180,000
Construct a small plant	100,000	20,000
Do nothing	200,000	0

Table 3.8

ALTERNATIVE	STATE OF NATURE: FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	MAXIMUM IN A ROW ($)
Construct a large plant	0	180,000	180,000
Construct a small plant	100,000	20,000	100,000
Do nothing	200,000	0	200,000

Minimax

Minimax criterion minimizes the maximum opportunity loss within each alternative

In-Class Example 1

- **Let's practice what we've learned. Use the decision table below to compute a choice using all the models**

Alternative	*State of Nature* Good Market ($)	Average Market ($)	Poor Market ($)
Construct a small plant	**75,000**	**25,000**	**-40,000**
Construct a large plant	**100,000**	**35,000**	**-60,000**
Do nothing	**0**	**0**	**0**

In-Class Example 1: Maximax

Alternative	*State of Nature* Good Market ($)	Average Market ($)	Poor Market ($)	*Maximum in a Row ($)*
Construct a small plant	**75,000**	**25,000**	**-40,000**	**75,000**
Construct a large plant	**100,000**	**35,000**	**-60,000**	**100,000**
Do nothing	**0**	**0**	**0**	**0**

In-Class Example 1: Maximin

Alternative	*State of Nature* Good Market ($)	Average Market ($)	Poor Market ($)	*Minimum in a Row ($)*
Construct a small plant	**75,000**	**25,000**	**-40,000**	**-40,000**
Construct a large plant	**100,000**	**35,000**	**-60,000**	**-60,000**
Do nothing	**0**	**0**	**0**	**0**

In-Class Example 1: Criterion of Realism

$\alpha = 0.6$

Alternative	State of Nature: Good Market ($)	Average Market ($)	Poor Market ($)	Criterion of Realism α=0.6 ($)
Construct a small plant	**75,000**	-	**-40,000**	**29,000**
Construct a large plant	**100,000**	-	**-60,000**	**36,000**
Do nothing	**0**	-	**0**	**0**

In-Class Example 1: Equally Likely

Alternative	State of Nature: Good Market ($)	Average Market ($)	Poor Market ($)	Row Average ($)
Construct a small plant	**75,000**	**25,000**	**-40,000**	**20,000**
Construct a large plant	**100,000**	**35,000**	**-60,000**	**25,000**
Do nothing	**0**	**0**	**0**	**0**

In-Class Example 1: Minimax Regret Opportunity Loss Table

Alternative	State of Nature: Good Market ($)	Average Market ($)	Poor Market ($)	Maximum Opp. Loss in a Row ($)
Construct a small plant	**25,000**	**10,000**	**40,000**	**40,000**
Construct a large plant	**0**	**0**	**60,000**	**60,000**
Do nothing	**100,000**	**35,000**	**0**	**100,000**

Decision Making Under Risk

- **Decision making when there are several possible states of nature and we know the probabilities associated with each possible state**
- **Most popular method is to choose the alternative with the highest *expected monetary value (EMV)***
- **EMV is the weighted sum of all possible payoffs for an alternative and is the long run average value for that decision**

EMV (alternative *i*) = (payoff of first state of nature)
x (probability of first state of nature)
+ (payoff of second state of nature)
x (probability of second state of nature)
+ … + (payoff of last state of nature)
x (probability of last state of nature)

EMV for Thompson Lumber

- Each market has a probability of 0.50
- Which alternative would give the highest EMV?

ALTERNATIVE	STATE OF NATURE: FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	EMV ($)
Construct a large plant	200,000	–180,000	?
Construct a small plant	100,000	–20,000	?
Do nothing	0	0	?
Probabilities	0.50	0.50	

Table 3.9

EMV for Thompson Lumber

- The calculations are

EMV (large plant) = (0.50)($200,000) + (0.50)(–$180,000)
= $10,000
EMV (small plant) = (0.50)($100,000) + (0.50)(–$20,000)
= $40,000
EMV (do nothing) = (0.50)($0) + (0.50)($0)
= $0

- The decision is to build a small plant, because it yields the maximum EMV

EMV for Thompson Lumber

ALTERNATIVE	STATE OF NATURE: FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	EMV ($)
Construct a large plant	200,000	–180,000	10,000
Construct a small plant	100,000	–20,000	40,000
Do nothing	0	0	0
Probabilities	0.50	0.50	

Table 3.9

Largest EMV

In-Class Example 2: Computing EMV

Using the table below to compute EMV

Alternative	*State of Nature*: *Good Market ($)*	*Average Market ($)*	*Poor Market ($)*
Construct a small plant	**75,000**	**25,000**	**-40,000**
Construct alarge plant	**100,000**	**35,000**	**-60,000**
Do nothing	**0**	**0**	**0**
Probability	**0.25**	**0.50**	**0.25**

In-Class Example 2: Computing EMV

- **The calculations are:**

EMV (small plant)
= (0.25)($75,000) + (0.50)($25,000) + (0.25)(–$40,000)
= $21,250
EMV (large plant)
= (0.25)($100,000) + (0.50)(35,000) + (0.25)(–$60,000)
= $27,500
EMV (do nothing)
= (0.25)($0) + (0.50)($0) + (0.25)($0)
= $0
Max. EMV = $27,500

In-Class Example 2: Computing EMV

Alternative	*State of Nature*			*EMV ($)*
	Good Market ($)	*Average Market ($)*	*Poor Market ($)*	
Construct a small plant	**75,000**	**25,000**	**-40,000**	**21,250**
Construct a large plant	**100,000**	**35,000**	**-60,000**	**27,500**
Do nothing	**0**	**0**	**0**	**0**
Probability	**0.25**	**0.50**	**0.25**	

Expected Value of Perfect Information (EVPI)

- **Marketing research company can find out what the exact outcome will be – perfect information – decision under risk → decision under certainty – with fee**
- **EVPI places an upper bound on what you should pay for additional information**

EVPI = EVwPI – Maximum EMV

- **EVwPI is the long run average return if we have perfect information before a decision is made (we need to compute it because we do not know it until after we pay)**

EVwPI = (best payoff for first state of nature)
x (probability of first state of nature)
+ (best payoff for second state of nature)
x (probability of second state of nature)
+ … + (best payoff for last state of nature)
x (probability of last state of nature)

Expected Value of Perfect Information (EVPI)

- **Scientific Marketing, Inc. offers analysis for Thompson Lumber Company that will provide certainty about market conditions (favorable or not)**
- **Additional information will cost $65,000**
- **Is it worth purchasing the information?**

Expected Value of Perfect Information (EVPI)

1. Best alternative for favorable state of nature is to build a large plant with a payoff of $200,000

   Best alternative for unfavorable state of nature is to do nothing with a payoff of $0 (see Table 3.9)

   EVwPI = ($200,000)(0.50) + ($0)(0.50) = $100,000

   We compute the best payoff for each outcome since we don't know what the research will tell us
2. The maximum EMV without additional information is $40,000

   EVPI = EVwPI – Maximum EMV
   = $100,000 – $40,000
   = $60,000

Expected Value of Perfect Information (EVPI)

1. Best alte… build a la… Best alte… to do no… EVwPI =

   So the maximum Thompson should pay for the additional information is $60,000
2. The maximum EMV without additional information is $40,000

   EVPI = EVwPI – Maximum EMV
   = $100,000 – $40,000
   = $60,000

In-Class Example 3: Computing EMV, EVwPI & EVPI

Using the table below to compute EMV, EVwPI and EVPI.

Alternative	State of Nature		
	Good Market ($)	Average Market ($)	Poor Market ($)
Construct a small plant	75,000	25,000	-40,000
Construct a large plant	100,000	35,000	-60,000
Do nothing	0	0	0
Probability	0.25	0.50	0.25

In-Class Example 3: Computing EMV, EVwPI & EVPI

Alternative	State of Nature			EMV
	Good Market ($)	Average Market ($)	Poor Market ($)	
Construct a small plant	75,000	25,000	-40,000	21,250
Construct a large plant	100,000	35,000	-60,000	27,500
Do nothing	0	0	0	0
Probability	0.25	0.50	0.25	

In-Class Example 3: Computing EMV, EVwPI & EVPI

EMV(small) = \$75,000*0.25 + \$25,000*0.5 + (–40,000*0.25) = \$21,250

EMV(large) = \$100,000*0.25 + \$35,000*0.5 + (–60,000*0.25) = \$27,500

EVwPI = \$100,000*0.25 + \$35,000*0.50 + 0*0.25 = \$42,500

Max. EMV = \$27,500

EVPI = EVwPI – max (*EMV*)
= \$42,500 – \$27,500 = \$15,000

Expected Opportunity Loss

- *Expected opportunity loss* (EOL) is the cost of not picking the best solution
- An alternative approach to maximizing EMV is to minimize EOL
- First construct an opportunity loss table
- For each alternative, multiply the opportunity loss by the probability of that loss for each possible outcome and add these together
- Minimum EOL will always result in the same decision as maximum EMV
- Minimum EOL will always equal EVPI

Expected Opportunity Loss

Payoff Table

ALTERNATIVE	STATE OF NATURE: FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)
Construct a large plant	200,000	–180,000
Construct a small plant	100,000	–20,000
Do nothing	0	0
Probabilities	0.50	0.50

Opportunity Loss Table

ALTERNATIVE	STATE OF NATURE: FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)
Construct a large plant	200,000 – 200,000	0 – (–180,000)
Construct a small plant	200,000 – 100,000	0 – (–20,000)
Do nothing	200,000 – 0	0 – 0
Probabilities	0.50	0.50

Expected Opportunity Loss

ALTERNATIVE	STATE OF NATURE: FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	EOL
Construct a large plant	0	180,000	90,000
Construct a small plant	100,000	20,000	60,000
Do nothing	200,000	0	100,000
Probabilities	0.50	0.50	

Table 3.10: Opportunity Loss Table

Minimum EOL = EVPI

EOL (large plant) = (0.50)(\$0) + (0.50)(\$180,000)
= \$90,000

EOL (small plant) = (0.50)(\$100,000) + (0.50)(\$20,000)
= \$60,000

EOL (do nothing) = (0.50)(\$200,000) + (0.50)(\$0)
= \$100,000

Sensitivity Analysis

- In the previous analyses (with the known payoffs and probabilities), we concluded that the best decision was to build a small plant
- What would happen if the values of payoff and probability changed
- Sensitivity analysis examines how our decision might change with different input data
- We investigate the impact of change in probability values

Sensitivity Analysis

- For the Thompson Lumber example

P = probability of a favorable market

$(1 - P)$ = probability of an unfavorable market

ALTERNATIVE	STATE OF NATURE: FAVORABLE MARKET ($)	STATE OF NATURE: UNFAVORABLE MARKET ($)	EMV ($)
Construct a large plant	200,000	–180,000	?
Construct a small plant	100,000	–20,000	?
Do nothing	0	0	?
Probabilities	*p*	1–*p*	

Sensitivity Analysis

EMV(Large Plant) $= \$200{,}000P - \$180{,}000*(1 - P)$
$= \$200{,}000P - \$180{,}000 + \$180{,}000P$
$= \$380{,}000P - \$180{,}000$

EMV(Small Plant) $= \$100{,}000P - \$20{,}000*(1 - P)$
$= \$100{,}000P - \$20{,}000 + \$20{,}000P$
$= \$120{,}000P - \$20{,}000$

EMV(Do Nothing) $= \$0P + 0*(1 - P)$
$= \$0$

Sensitivity Analysis

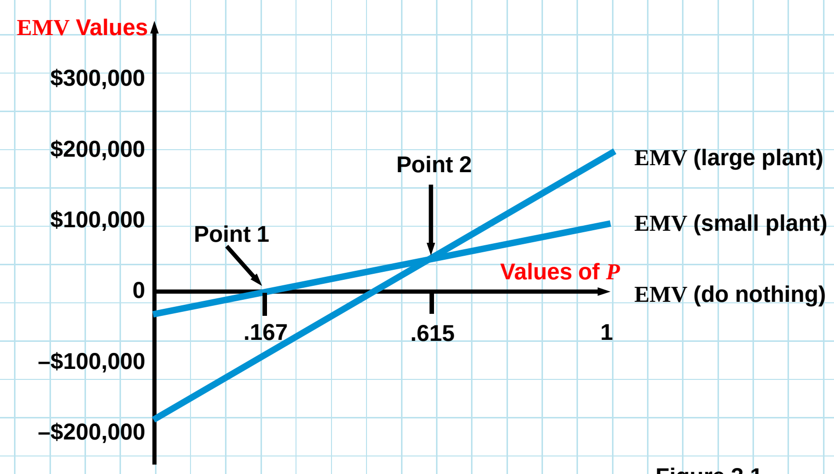


Figure 3.1

Sensitivity Analysis

Point 1:

EMV(do nothing) = EMV(small plant)

$$0 = \$120{,}000P - \$20{,}000 \qquad P = \frac{20{,}000}{120{,}000} = 0.167$$

Point 2:

EMV(small plant) = EMV(large plant)

$$\$120{,}000P - \$20{,}000 = \$380{,}000P - \$180{,}000$$

$$P = \frac{160{,}000}{260{,}000} = 0.615$$

Sensitivity Analysis

RANGE OF P VALUES	BEST ALTERNATIVE
Less than 0.167	Do nothing
0.167 – 0.615	Construct a small plant
Greater than 0.615	Construct a large plant

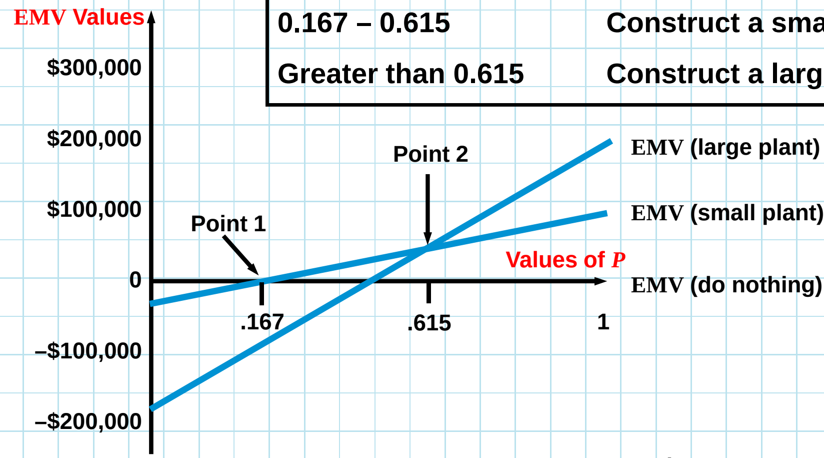


Figure 3.1

Using Excel QM to Solve Decision Theory Problems

Compute the EMV for each alternative using the SUMPRODUCT function, the worst case using the MIN function, and the best case using the MAX function.

To calculate the EVPI, find the best outcome for each scenario.

	A	B	C	D	E	F	G	H	I
1	Thompson Lumber								
2									
3	Decision Tables								
4	Enter the profits or costs in the main body of the data table. Enter probabilities in the first row if you want								
5	to compute the expected value.								
6	Data				Results				
7	Profit	Favorable Market	Unfavorable Market		EMV	Minimum	Maximum		Hurwicz
8	Probability	0.5	0.5					c	0.8
9	Large Plant	200000	-180000		=SUMPRODUCT(B$8:C$8,B9:C9)	=MIN(B9:C9)	=MAX(B9:C9)		=$I$8*G9+(1-$I$8)*F9
10	Small plant	100000	-20000		=SUMPRODUCT(B$8:C$8,B10:C10)	=MIN(B10:C10)	=MAX(B10:C10)		=$I$8*G10+(1-$I$8)*F10
11	Do nothing	0	0		=SUMPRODUCT(B$8:C$8,B11:C11)	=MIN(B11:C11)	=MAX(B11:C11)		
12				M	=MAX(E9:E11)	=MAX(F9:F11)	=MAX(G9:G11)		=MAX(I9:I11)
13									
14	Expected Value of Perfect Information								
15	Column best	=MAX(B9:B11)	=MAX(C9:C11		=SUMPRODUCT(B$8:C$8,B15:C15)	<-Expected value under certainty			
16					=E12	<-Best expected value			
17					=E15-E12	<-Expected value of perfect information			
18									
19	Regret								
20		=B7	=C7		Expected	Maximum			
21	=A8	=B8	=C8						
22	=A9	=B15 - B9	=C15 - C9		=SUMPRODUCT(B$8:C$8,B22:C22)	=MAX(B22:C22)			
23	=A10	=B15 - B10	=C15 - C10		=SUMPRODUCT(B$8:C$8,B23:C23)	=MAX(B23:C23)			
24	=A11	=B15 - B11	=C15 - C11		=SUMPRODUCT(B$8:C$8,B24:C24)	=MAX(B24:C24)			
25				M	=MIN(E22:E24)	=MIN(F22:F24)			
26									
27									
28									
29									

Find the best outcome for each measure using the MAX function.

Use SUMPRODUCT to compute the product of the best outcomes by the probabilities and find the difference between this and the best expected value yielding the EVPI.

Program 3.1A

Using Excel QM to Solve Decision Theory Problems

	A	B	C	D	E	F	G	H	I
1	Thompson Lumber								
2									
3	Decision Tables								
4	Enter the profits or costs in the main body of the data table. Enter probabilities in the								
5	first row if you want to compute the expected value.								
6	Data				Results				
7	Profit	Favorable Market	Unfavorable Market		EMV	Minimum	Maximum		Hurwicz
8	Probability	0.5	0.5					coefficient	0.8
9	Large Plant	200000	-180000		10000	-180000	200000		124000
10	Small plant	100000	-20000		40000	-20000	100000		76000
11	Do nothing	0	0		0	0	0		
12				Maximum	40000	0	200000		124000
13									
14	Expected Value of Perfect Information								
15	Column best	200000	0		100000	<-Expected value under certainty			
16					40000	<-Best expected value			
17					60000	<-Expected value of perfect information			
18									
19	Regret								
20		Favorable M	Unfavorable Market		Expected	Maximum			
21	Probability	0.5	0.5						
22	Large Plant	0	180000		90000	180000			
23	Small plant	100000	20000		60000	100000			
24	Do nothing	200000	0		100000	200000			
25				Minimum	60000	100000			

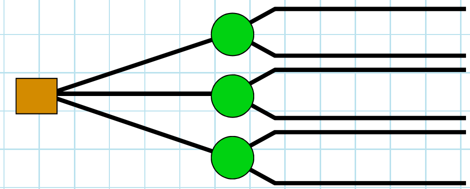
Program 3.1B

Decision Trees

- Any problem that can be presented in a decision table can also be graphically represented in a *decision tree*
- Decision trees are most beneficial when a sequence of decisions must be made
- All decision trees contain *decision points* or *decision nodes* and *state-of-nature points* or *state-of-nature nodes*
  - A decision node – from which one of several alternatives may be chosen
  - A state-of-nature node – out of which one state of nature will occur

Structure of Decision Trees

- Trees start from left to right
- Represent decisions and outcomes in sequential order
- Squares represent decision nodes
- Circles represent state of nature nodes
- Lines or branches connect the decision and the state of nature nodes



Thompson's Decision Tree

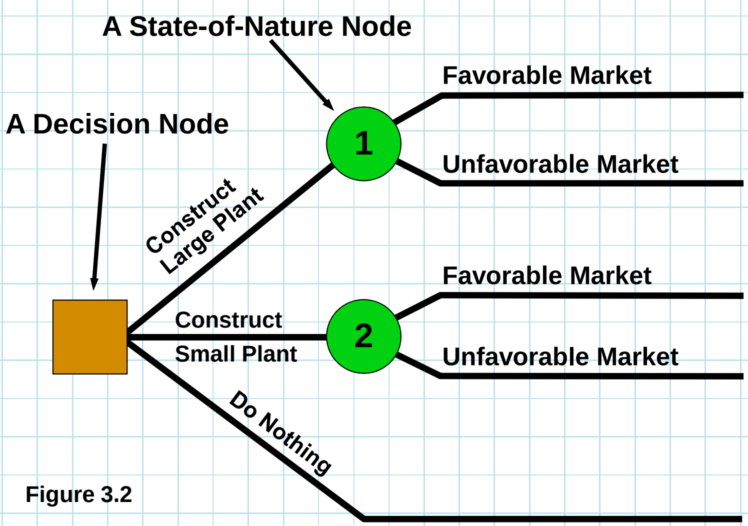


Figure 3.2

Five Steps to Decision Tree Analysis

1. Define the problem
2. Structure or draw the decision tree
3. Assign probabilities to the states of nature
4. Estimate payoffs for each possible combination of alternatives and states of nature
5. Solve the problem by computing expected monetary values (EMVs) for each state of nature node by working backward (from right to left) and select the best EMV at each decision node

Thompson's Decision Tree

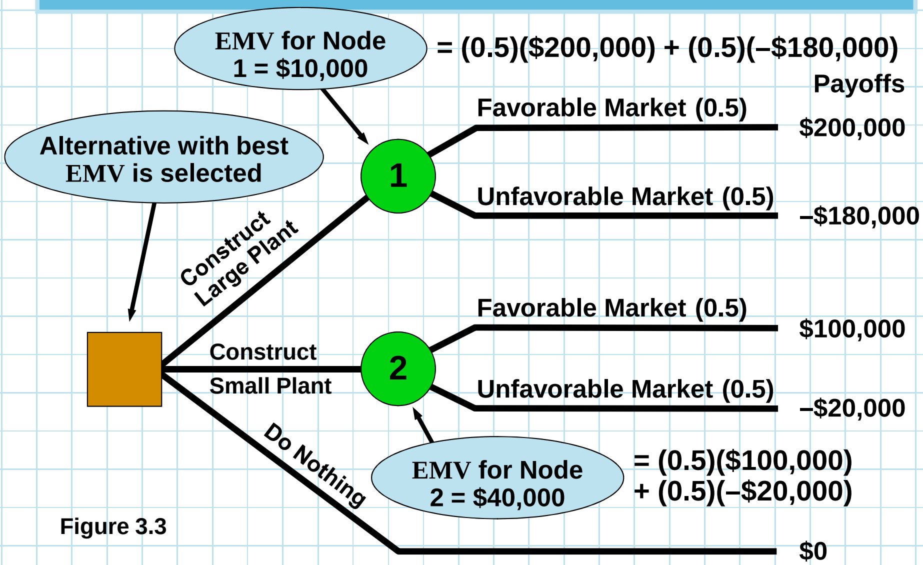


Figure 3.3

Thompson's Complex Decision Tree

- Decision trees are much more powerful tools than decision tables when a sequence of decisions need to be made
- Suppose Thompson Lumber has to make two decisions, with the second decision dependent upon the outcome of the first
  1) Whether or not to conduct a market survey at the cost of $10,000
  2) Whether to build a large, small or no plant

Thompson's Complex Decision Tree

- The first decision will help Thompson Lumber to make the second decision – which alternative to pursue (large, small or no plant)
- The survey does not provide perfect information, but it will help
- The cost of survey must be deducted from the original payoffs in decision tree analysis

Thompson's Complex Decision Tree

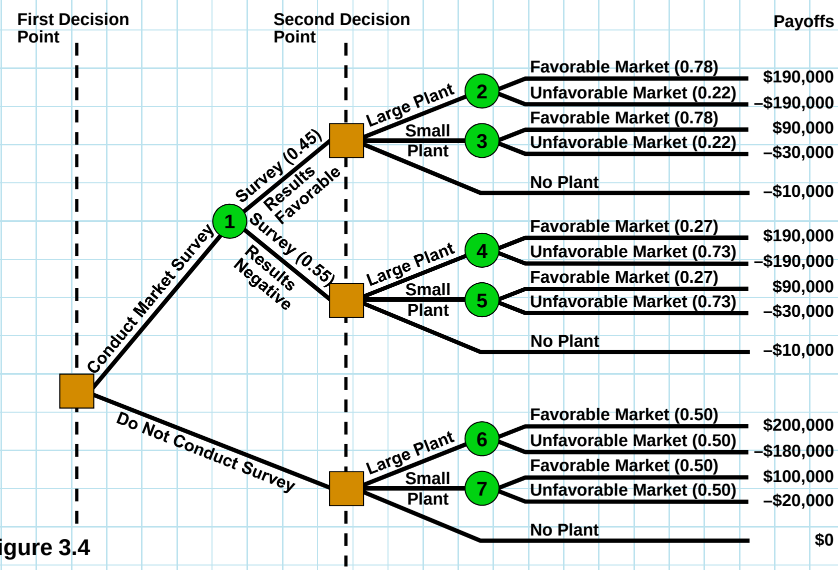


Figure 3.4

Thompson's Complex Decision Tree

1. **Given favorable survey results,**

EMV(node 2) = EMV(large plant | positive survey)
= (0.78)($190,000) + (0.22)(–$190,000) = $106,400

EMV(node 3) = EMV(small plant | positive survey)
= (0.78)($90,000) + (0.22)(–$30,000) = $63,600

EMV for no plant = –$10,000

2. **Given negative survey results,**

EMV(node 4) = EMV(large plant | negative survey)
= (0.27)($190,000) + (0.73)(–$190,000) = –$87,400

EMV(node 5) = EMV(small plant | negative survey)
= (0.27)($90,000) + (0.73)(–$30,000) = $2,400

EMV for no plant = –$10,000

Thompson's Complex Decision Tree

3. **Compute the expected value of the market survey,**

EMV(node 1) = EMV(conduct survey)
= (0.45)($106,400) + (0.55)($2,400)
= $47,880 + $1,320 = $49,200

4. **If the market survey is not conducted,**

EMV(node 6) = EMV(large plant)
= (0.50)($200,000) + (0.50)(–$180,000) = $10,000

EMV(node 7) = EMV(small plant)
= (0.50)($100,000) + (0.50)(–$20,000) = $40,000

EMV for no plant = $0

5. **Best choice is to seek marketing information**

Construct a large plant if survey results are favorable

Construct a small plant if survey results are negative

Thompson's Complex Decision Tree

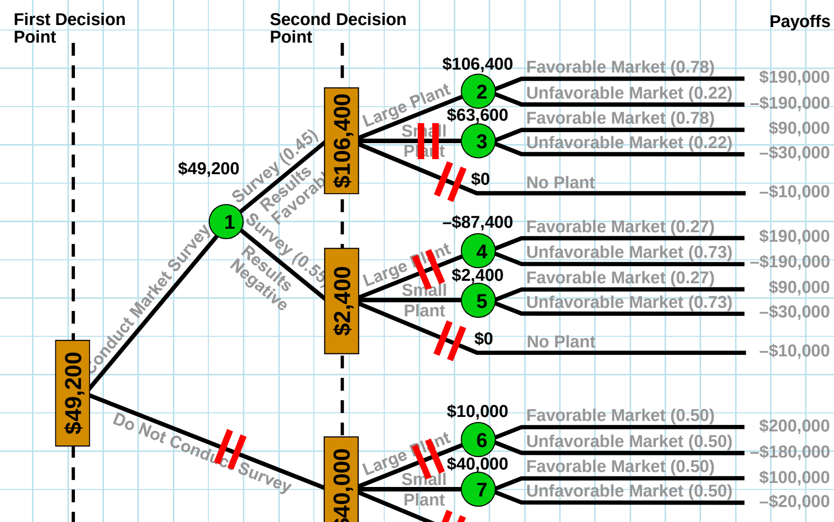


Figure 3.4

Expected Value of Sample Information

- **Thompson wants to know the actual value of doing the survey – it is not free**

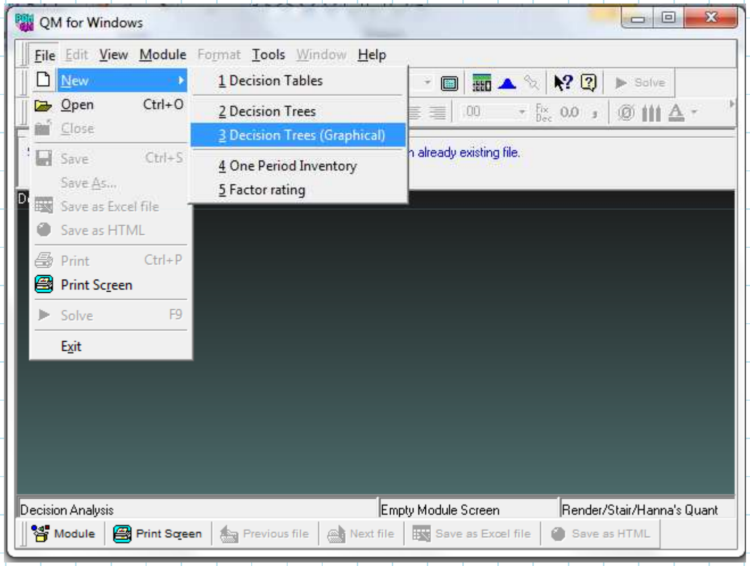
EVSI = (Expected value *with* sample information, assuming no cost to gather it) – (Expected value of best decision *without* sample information)

= (EV with sample information + cost) – (EV without sample information)

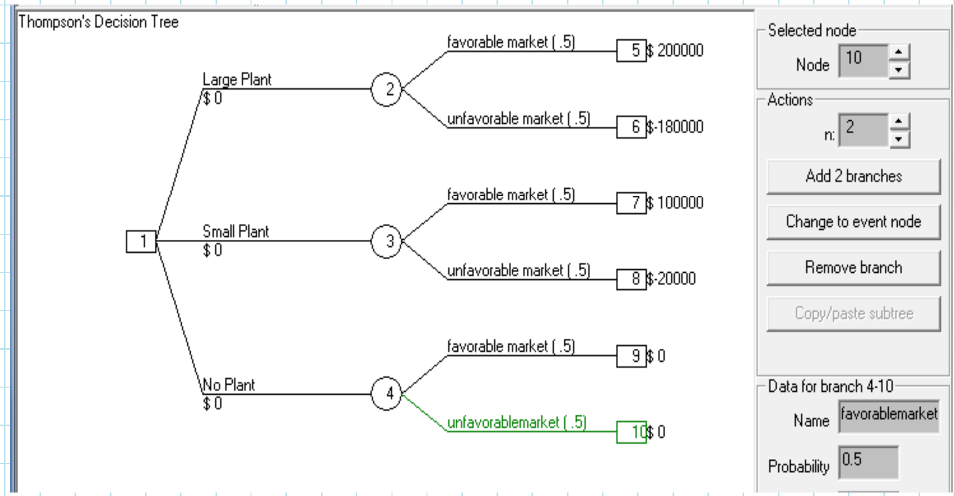
EVSI = ($49,200 + $10,000) – $40,000 = $19,200

Thompson could pay up to $19,200 for the survey

Using Excel QM to Draw Decision Trees



Using Excel QM to Draw Decision Trees



Sensitivity Analysis

- As with decision tables, sensitivity analysis can be applied to decision trees as well
- How sensitive are the decisions to changes in the problem parameters ?
  - Consider how sensitive our decision is to the probability of a favorable survey result?
  - That is, if the probability of a favorable result (p = .45) were to change, would we make the same decision?
  - How much could it change before we would make a different decision?

Sensitivity Analysis

p = probability of a favorable survey result

$(1 – p)$ = probability of a negative survey result

$$\text{EMV(node 1)} = (\$106{,}400)p + (\$2{,}400)(1 - p)$$
$$= \$104{,}000p + \$2{,}400$$

We are indifferent when the EMV of node 1 is the same as the EMV of not conducting the survey, $40,000

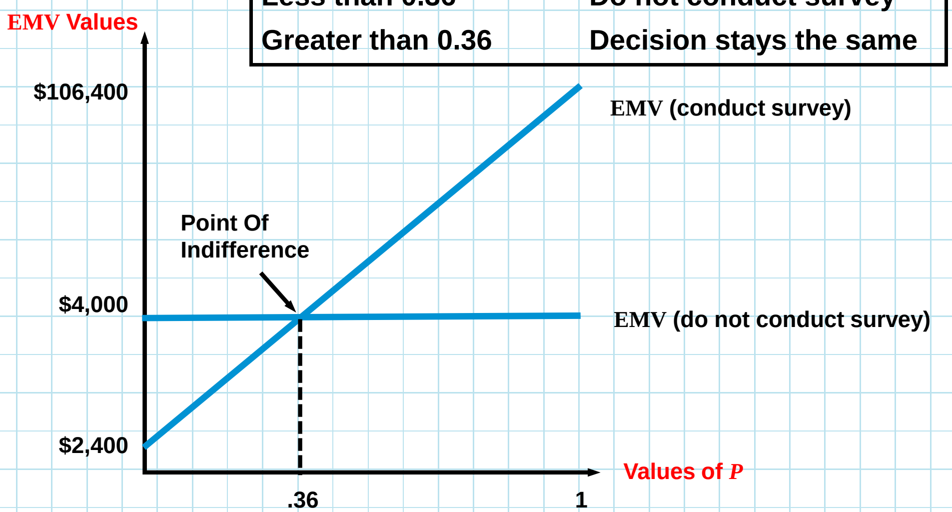
$$\$104{,}000p + \$2{,}400 = \$40{,}000$$
$$\$104{,}000p = \$37{,}600$$
$$p = \$37{,}600/\$104{,}000 = 0.36$$

So, if $p < 0.36$, do not conduct the survey

if $p > 0.36$, the decision will stay the same

Sensitivity Analysis

RANGE OF *P* VALUES	BEST ALTERNATIVE
Less than 0.36	Do not conduct survey
Greater than 0.36	Decision stays the same



Utility Theory

- Monetary value (EMV) is not always a true indicator of the overall value of the result of a decision
  - A person may settle a lawsuit out of court even though they may get more by going to trial and winning
  - A businessperson may rule out a potential decision because it could bankrupt the company if things go bad even though the expected return is better than that of all the other alternatives
- The overall value of a decision is called ***utility***
- Rational people make decisions to maximize their utility

Utility Theory

- Suppose you bought a winning lottery ticket of $2 million. To make the game more exciting, a fair coin would be flipped. If it is tail, you would win $5 million. If it is head, you would loss $2 million. A wealthy businessman offered you $2 million for the ticket. What would you do, sell it or hold on to it ? Why ?

Utility Theory

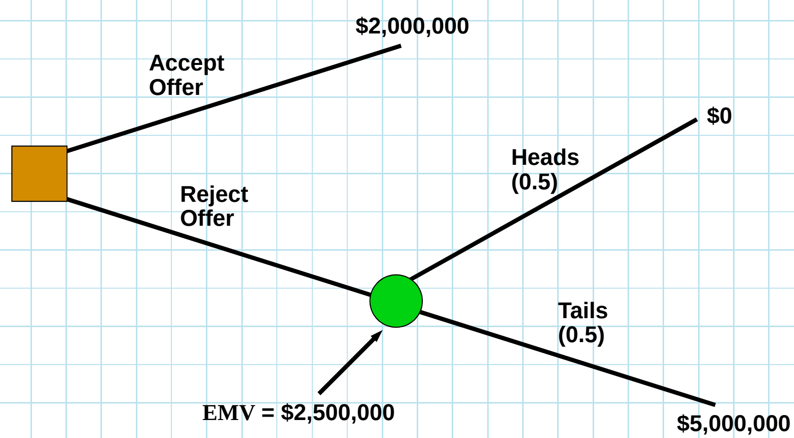


Figure 3.6

Utility Theory

- The EMV of rejecting the offer and continue with the game is higher than accepting the offer and go with $2,000,000.
- However many people would take $2,000,000 (or even less) rather than flip the coin even though the EMV says otherwise.
- People have different feelings about seeking or avoiding risk

Utility Theory

- When an extremely large payoff or loss is involved, EMV may not always be the only criterion for making decisions
- One way to incorporate your own attitude toward risk is through utility theory
- We first show how to measure utility and then show how to use utility to make decision

Utility Theory

- The first step is to assign utility values to each monetary value in a given situation
  - *Utility assessment* usually assigns the worst outcome a utility of 0, and the best outcome, a utility of 1
  - All other outcomes have a value between 0 and 1
- A *standard gamble* is used to determine utility values (Fig. 3.7)
  - *p* is the probability of obtaining the best outcome and (1-*p*) the worst outcome

Utility Theory

- Assessing the utility of any other outcome involves determining the probability (*p*) which makes you indifferent between alternative 1 (gamble between the best and worst outcome) and alternative 2 (obtaining the other outcome for sure)
- When you are *indifferent* between alternatives 1 and 2, the expected utilities for these two alternatives must be equal

Expected utility of alternative 2 = Expected utility of alternative 1

Utility of other outcome = (p)(utility of best outcome, which is 1) + (1 – p)(utility of the worst outcome, which is 0)

= (p)(1) + (1 – p)(0) = p

Standard Gamble

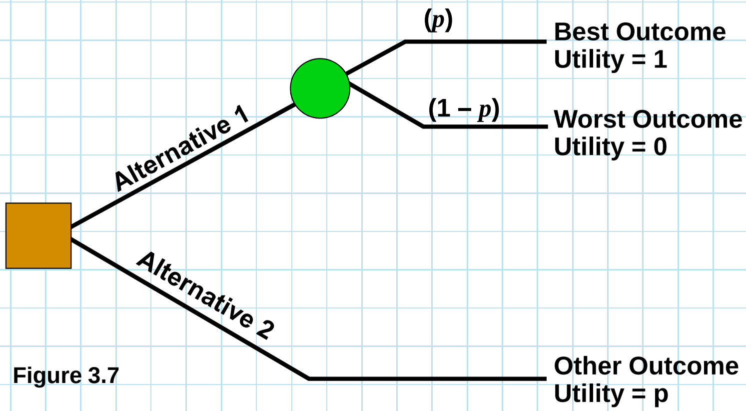


Figure 3.7

Utility of other outcome = (p)(1) + (1-p)(0) = p

Find the value of p that makes you indifferent between alternatives 1 and 2 (this is totally subjective)

Standard Gamble Example

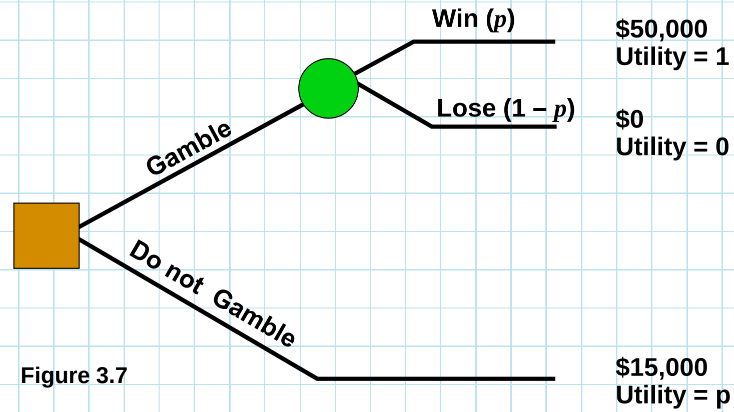


Figure 3.7

Find the probability value of p that makes you indifferent between gambling and not gambling (this is totally subjective)

Standard Gamble Example

- Suppose you are willing to give up the guaranteed payoff of $15,000 and gamble for the $50,000, if the probability of winning is 50%.
- You are indifferent between gambling and not gambling when p = 50%
- $U(\$15{,}000) = p = 0.5$
- $EMV(\text{gamble}) = 0.5 \times \$50{,}000 + (1–0.5) \times \$0$
  $= \$25{,}000$
- From the utility perspective, the expected value of gambling is only $15,000

Investment Example

- Jane Dickson is considering a real estate investment. It will pay $10,000 in a good market or $0 in a bad market.
- Jane could also leave her money in the bank and earn a $5,000 return.
- Unless there is an 80% chance of getting $10,000 from the real estate deal, Jane would prefer to put the money in the bank.
- So if $p = 0.80$, Jane is indifferent between the bank or the real estate investment
- Thus, Jane's utility for $5,000 is 80% which is the same as the value of p.

Investment Example

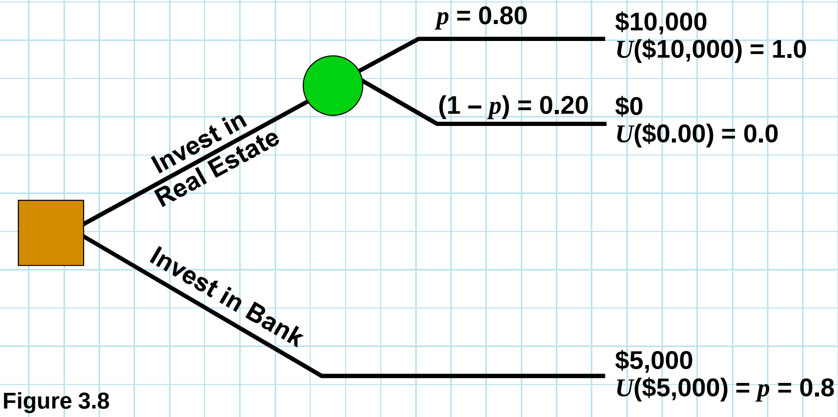


Figure 3.8

Utility for $5,000 = U($5,000) = p = 0.8

Investment Example

- We can assess other utility values in the same way
- For Jane these are

Utility for $7,000 = 0.90
Utility for $3,000 = 0.50

- Jane Dickson wants to construct a utility curve revealing her preference for money between $0 and $10,000 under the risk
- A utility curve plots the utility value versus the monetary value
- Using the three utilities for different dollar amounts, it is enough to construct a utility curve assessing Jane's feeling toward risk

Utility Curve

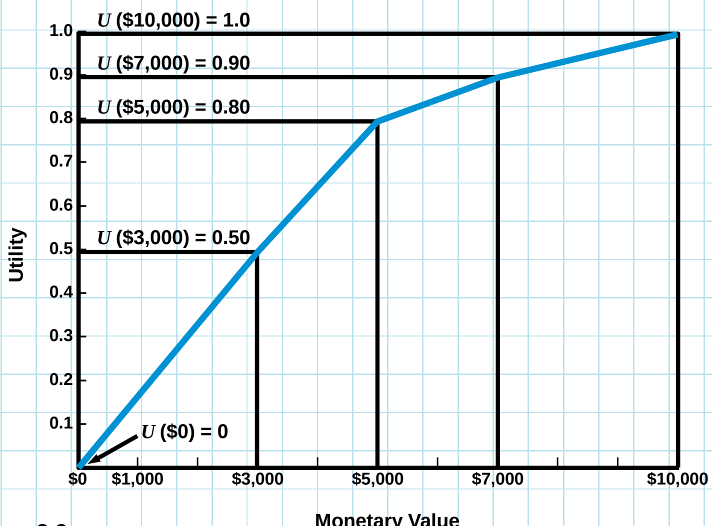


Figure 3.9

Utility Curve

- Jane's utility curve is typical of a risk avoider
  - A risk avoider gets less utility from greater risk
  - Avoids situations where high losses might occur
  - As monetary value increases, the utility curve increases at a slower rate
  - A risk seeker gets more utility from greater risk
  - As monetary value increases, the utility curve increases at a faster rate
  - Someone who is indifferent will have a linear utility curve – so he/she can use EMV to make decisions

Utility Curve

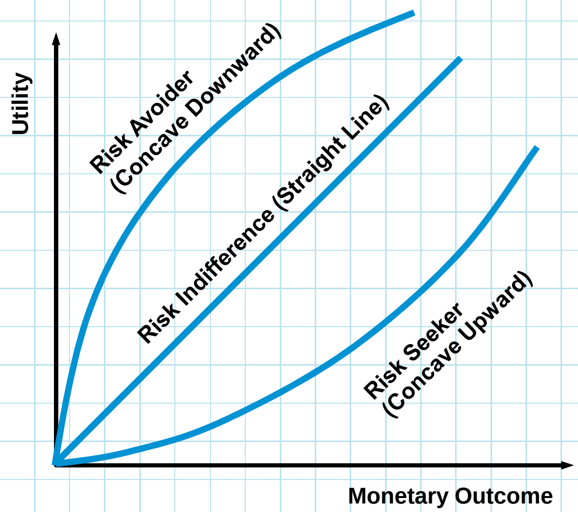


Figure 3.10

Utility as a Decision-Making Criteria

- Once a utility curve has been developed it can be used in making decisions
- Replace monetary outcomes with utility values
- The expected utility is computed instead of the EMV

Utility as a Decision-Making Criteria

- Mark Simkin loves to gamble
- He plays a game tossing thumbtacks in the air
- If the thumbtack lands point up, Mark wins $10,000

- If the thumbtack lands point down, Mark loses $10,000
- Should Mark play the game (alternative 1) or should he not play the game (alternative 2) ?
- Mark has $20,000 to gamble

Utility as a Decision-Making Criteria

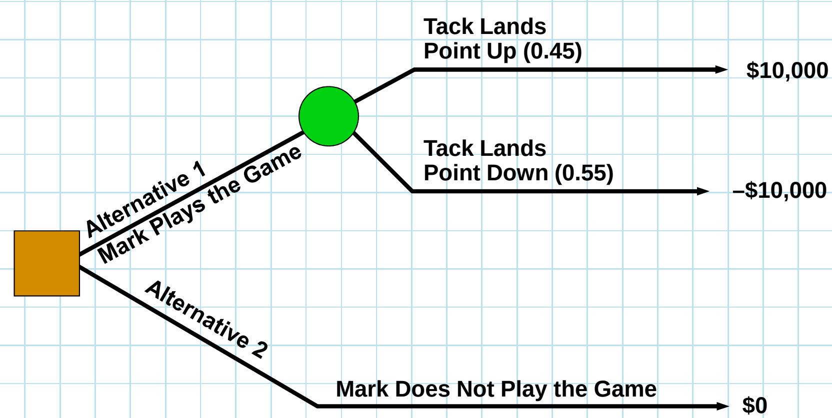


Figure 3.11

EMV (Alt. 1) = 0.45×$10,000+0.55 ×$–10,000 = –$1,000

Utility as a Decision-Making Criteria

- **Step 1 – Define Mark's utilities (Fig. 3.12)**

U (–\$20,000) = 0.00
U (–\$10,000) = 0.05
U (\$0) = 0.15
U (\$10,000) = 0.30
U (\$20,000) = 1.00

- **Step 2 – Replace monetary values with utility values (Fig. 3.13)**

E(alternative 1: play the game) = (0.45)(0.30) + (0.55)(0.05)
= 0.135 + 0.027 = 0.162

E(alternative 2: don't play the game) = 0.15

Utility as a Decision-Making Criteria

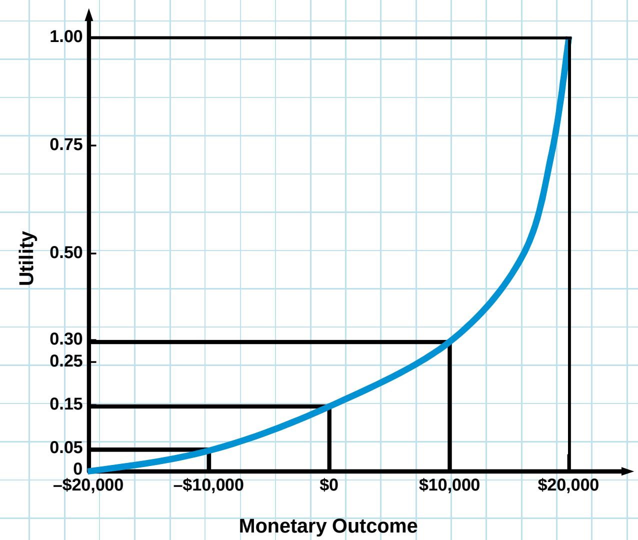


Figure 3.12

Utility as a Decision-Making Criteria

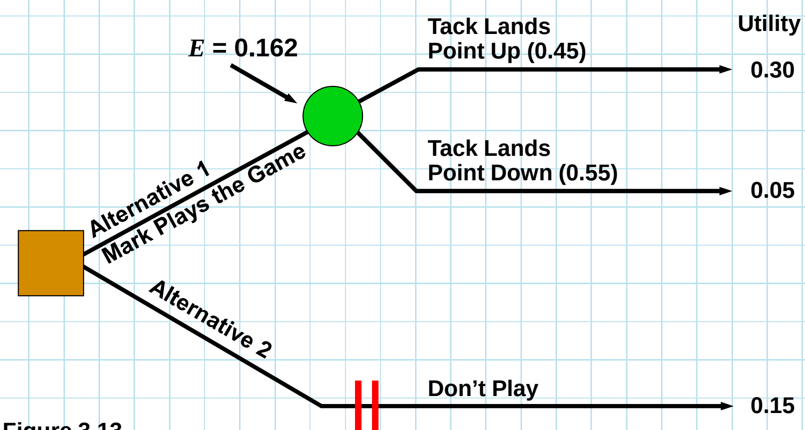


Figure 3.13

Alternative 1 is the best choice using utility approach. If EMV is used, alternative 2 is the best decision.

Homework Assignment

http://www.sci.brooklyn.cuny.edu/~dzhu/busn3430/