Homework #2

Chapter 14

1. Put the following predicate calculus expression in clause form:

   $\forall$ (X) (p(X) $\rightarrow$ { $\forall$ (Y) [p(Y) $\rightarrow$ p(f(X, Y))] $\wedge \neg \forall$ (Y) [q(X, Y) $\rightarrow$ p(Y)]})

Solution:

1. Eliminate conditionals ($\rightarrow$) using: **a $\rightarrow$ b $\equiv \neg$ a $\vee$ b**

   $\forall$ (X) ($\neg$ p(X) $\vee$ { $\forall$ (Y) [$\neg$ p(Y) $\vee$ p(f(X, Y))] $\wedge \neg \forall$ (Y) [$\neg$ q(X, Y) $\vee$ p(Y)]})

2. When possible, eliminate negations or reduce their scope using

   $\neg$ ($\neg$ a) $\equiv$ a
   $\neg$ (a $\vee$ b) $\equiv \neg$ a $\wedge \neg$ b $\qquad$ $\neg$ ($\exists$X) a(X) $\equiv$ ($\forall$X) $\neg$ a(X)
   $\neg$ (a $\wedge$ b) $\equiv \neg$ a $\vee \neg$ b $\qquad$ $\neg$ ($\forall$X) a(X) $\equiv$ ($\exists$X) $\neg$ a(X)

   $\forall$ (X) ($\neg$ p(X) $\vee$ { $\forall$ (Y) [$\neg$ p(Y) $\vee$ p(f(X, Y))] $\wedge \exists$ (Y) [q(X, Y) $\wedge \neg$ p(Y)]})

3. Standardize variables (each quantifier has unique variable name)

   $\forall$ (X) ($\neg$ p(X) $\vee$ { $\forall$ (Y) [$\neg$ p(Y) $\vee$ p(f(X, Y))] $\wedge \exists$ (Z) [q(X, Z) $\wedge \neg$ p(Z)]})

4. Move all $\forall$ to the front without changing their order

   $\forall$ (X) $\forall$ (Y) ($\neg$ p(X) $\vee$ { [$\neg$ p(Y) $\vee$ p(f(X, Y))] $\wedge \exists$ (Z) [q(X, Z) $\wedge \neg$ p(Z)]})

5. Eliminate existential quantifiers using Skolem functions

   Since the existential quantified Z is within the scope of universal quantified X (not Y), we replace Z with a skolem function g(X):

   $\forall$ (X) $\forall$ (Y) ($\neg$ p(X) $\vee$ { [$\neg$ p(Y) $\vee$ p(f(X, Y))] $\wedge$ [q(X, g(X)) $\wedge \neg$ p(g(X))]})

6. Drop the universal quantifiers as necessary

   $\neg$ p(X) $\vee$ { [$\neg$ p(Y) $\vee$ p(f(X, Y))] $\wedge$ [q(X, g(X)) $\wedge \neg$ p(g(X))]}

7. Convert the expression to conjunctive normal form using: $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

$\Rightarrow$ $\neg$ p(X) $\vee$ { [$\neg$ p(Y) $\vee$ p(f(X, Y))] $\wedge$ [q(X, g(X)) $\wedge$ $\neg$ p(g(X))]}

$\Rightarrow$ {$\neg$ p(X) $\vee$ [$\neg$ p(Y) $\vee$ p(f(X, Y))]} $\wedge$ {$\neg$ p(X) $\vee$ [q(X, g(X)) $\wedge$ $\neg$ p(g(X))]}

$\Rightarrow$ [$\neg$ p(X) $\vee$ $\neg$ p(Y) $\vee$ p(f(X, Y))] $\wedge$ {[$\neg$ p(X) $\vee$ q(X, g(X))] $\wedge$ [$\neg$ p(X) $\vee$ $\neg$ p(g(X))]}

$\Rightarrow$ [$\neg$ p(X) $\vee$ $\neg$ p(Y) $\vee$ p(f(X, Y))] $\wedge$ [$\neg$ p(X) $\vee$ q(X, g(X))] $\wedge$ [$\neg$ p(X) $\vee$ $\neg$ p(g(X))]

8. Eliminate $\wedge$ signs by writing the expression as a set of clauses

$\neg$ p(X) $\vee$ $\neg$ p(Y) $\vee$ p(f(X, Y))

$\neg$ p(X) $\vee$ q(X, g(X))

$\neg$ p(X) $\vee$ $\neg$ p(g(X))

9. Rename variables in clauses so that each clause has unique variable name

$\neg$ p(X) $\vee$ $\neg$ p(Y) $\vee$ p(f(X, Y))

$\neg$ p(U) $\vee$ q(U, g(U))

$\neg$ p(V) $\vee$ $\neg$ p(g(V))

2. Prove the following using resolution refutation method. Draw an inversed binary tree to show the resolution process.

Fact: $\neg d(f) \vee [b(f) \wedge c(f)]$

Rules: $\neg d(X) \rightarrow \neg a(X)$

$b(Y) \rightarrow e(Y)$

$g(W) \rightarrow c(W)$

Prove: $\neg a(Z) \vee e(Z)$

Solution:

First rewrite the above logical expressions in clause form:

Fact: $\neg d(f) \vee b(f)$

$\neg d(f) \vee c(f)$

Rules: $d(X) \vee \neg a(X)$

$\neg b(Y) \vee e(Y)$

$\neg g(W) \vee c(W)$

Negated Goal: $a(Z)$

$\neg e(Z)$

The following binary tree shows the proof process: