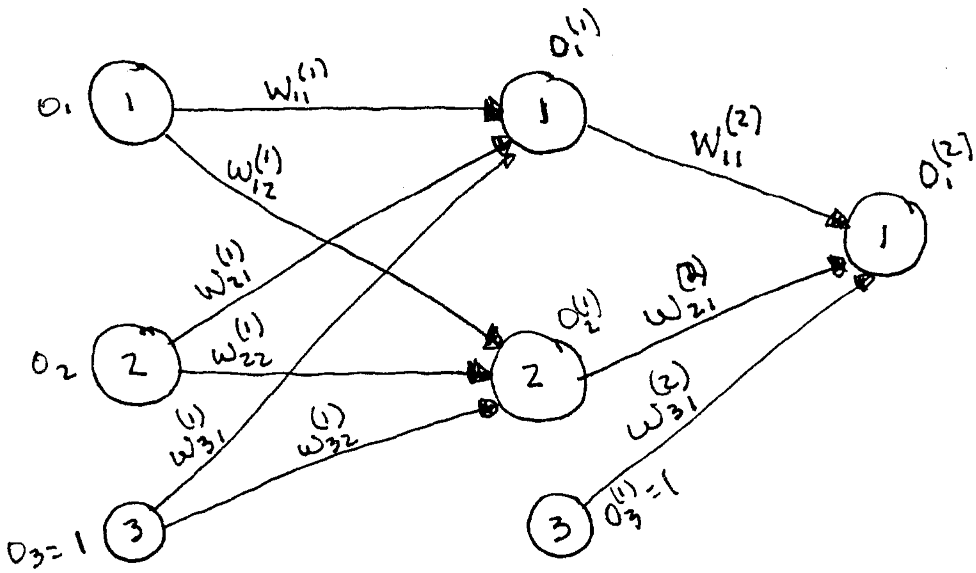
VI-B

Note: Check the arithmetic.

Three-layer network for solving the XOR problem



Initial weights and threshold levels are set randomly:

$w_{11}^{(1)} = 0.5 \quad (w_{13})$

$w_{12}^{(1)} = 0.9 \quad (w_{14})$

$w_{21}^{(1)} = 0.4 \quad (w_{23})$

$w_{22}^{(1)} = 1.0 \quad (w_{24})$

$w_{11}^{(2)} \;\; -1.2 \quad (w_{35})$

$w_{21}^{(2)} = 1.1 \quad (w_{45})$

$w_{31}^{(1)} = 0.8 \quad (\theta_3)$

$w_{32}^{(1)} = -0.1 \quad (\theta_4)$

$w_{31}^{(2)} = 0.3 \quad (\theta_5)$

VI-B

Let $O_1 = O_2 = 1$
and target output $= 0$.

Feedforward Computation

Calculate the outputs at the hidden layer $\bar{o}^{(1)}$

$$O_1^{(1)} = \text{sigmoid}\left(O_1 w_{11}^{(1)} + O_2 w_{21}^{(1)} - w_{31}^{(1)}\right)$$

$$= \frac{1}{\left[1 + e^{-(1\times 0.3 + 1\times 0.4 - 1\times 0.8)}\right]} = \underline{0.5250}$$

$$O_2^{(1)} = \text{sigmoid}\left(O_1 w_{12}^{(1)} + O_2 w_{22}^{(1)} - w_{32}^{(1)}\right)$$

$$= \frac{1}{\left[1 + e(-(1\times 0.9 + 1\times 1.0 + 1\times 0.1)\right]} = \underline{0.8808}$$

Next, calculate the output of the neural net itself:

$$O_1^{(2)} = \text{Sigmoid}\left(O_1^{(1)} \times w_{11}^{(2)} + O_2^{(1)} \times w_{21}^{(2)} - w_{31}^{(2)}\right)$$

$$= \frac{1}{\left[1 + e^{-(0.5250\times 1.2 + 0.8808\times 1.1 - 1\times 0.3}\right]}$$

$$= \underline{0.5097}$$

VI - B

The error obtained $\boxed{e = o_1^{(2)} - t_1}$

well calculated as

$$\boxed{e = t_1 - o_1^{(2)}}$$

our text, adds - sign on updates!

$$e = 0 - 0.5097 = -0.5097$$

$$\boxed{e = 0.5097 - 0 = 0.5097}$$

Backpropagation to the output layer

Backpropagated error at the output

$$\delta_1^{(2)} = o_1^{(2)} (1 - o_1^{(2)}) (o_1^{(2)} - t_1)$$

$$= 0.5097 \times (1 - 0.5097) \times (0.5097)$$

$$= 0.1274$$

VI-B

Next, calculate the <u>weight corrections</u> <u>at output units</u>, $\gamma = 0.1$ learning rate.

$$\Delta w_{ij}^{(2)} = -\gamma\, O_i^{(1)} \delta_1^{(2)} \quad \text{for } i = 1, \dots 3$$

$j = 1$ // only one output.

$$\Delta w_{11}^{(2)} = (-0.1) \times 0.5250 \times (0.1274) = -0.0067$$

$$\Delta w_{21}^{(2)} = (-0.1) \times 0.8808 \times (0.1274) = -0.0112$$

$$\Delta w_{31}^{(2)} = (-0.1) \times (-1) \times (0.1274) = -0.0127$$

VI-B

Now, we calculate the backpropagated error at the hidden layer:

$$\delta_j^{(1)} = O_j^{(1)}\left(1 - O_j^{(1)}\right) \sum_{q=1}^{m} w_{jq}^{(2)} \delta_q^{(2)}$$

$$\delta_1^{(1)} = O_1^{(1)}\left(1 - O_1^{(1)}\right) \times w_{11}^{(2)} \times \delta_1^{(2)}$$

$$= 0.5250 \times (1 - 0.5250) \times (-1.2) \times (.1274)$$

$$= \underline{-0.0381}$$

$$\delta_2^{(1)} = O_2^{(1)}\left(1 - O_2^{(1)}\right) \times w_{21}^{(2)} \times \delta_1^{(2)}$$

$$= 0.5250 \times (1 - 0.5250) \times 1.1 \times (.1274)$$

$$= +0.0147$$

VT-13

Weight corrections = ?

$$\Delta w_{11}^{(1)} = -\gamma \times O_1^{(1)} \times \delta_1^{(1)}$$
$$= (-0.1) \times 1 \times (-0.0381)$$
$$= \underline{0.0038}$$

$$\Delta w_{12}^{(1)} = -\gamma \times O_1^{(1)} \times \delta_2^{(1)}$$
$$= (-0.1) \times 1 \times (0.0147)$$
$$= \underline{-.0015}$$

$$\Delta w_{21}^{(1)} = -\gamma \times O_2^{(1)} \times \delta_1^{(1)}$$
$$= (-0.1) \times 1 \times (-0.0381)$$
$$= \underline{0.0038}$$

$$\Delta w_{22}^{(1)} = -\gamma \times O_2^{(1)} \times \delta_2^{(1)}$$
$$= (-0.1) \times 1 \times (0.0147)$$
$$= \underline{-0.0015}$$

(θ_3) $$\Delta w_{31}^{(1)} = -\gamma \times (1) \times \delta_1^{(1)}$$
$$= (-0.1) \times (-1)^{?} \times (-0.0381)$$
$$= \underline{-0.0038}$$

(θ_4) $$\Delta w_{32}^{(1)} = -\gamma \times (1) \times \delta_2^{(1)}$$
$$= (-0.1) \times -1^{?} \times (0.0147)$$
$$= \underline{0.0015}$$