Exam 3, Mathematics 11.1, Section TF10 43 copies
10:50–12:00 am, May 2, 2008, 330NE
Instructor: Attila Máté

Follow these instructions carefully:

Work on the paper provided; do not use your own paper. *Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet).* On the top of each page, *print* your name (*encircle your last name*) and indicate the number of the problem you are working on by writing e.g. *"Problem #4"*. Always *encircle* your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing *"Answer to Part b)"* (the number of the problem should be on the top of the page). Do not use a *red* pen or a *red* pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the *same* amount of credit.

1. *a*) State (without proof) the Mean-Value Theorem (for differentiation).

b) Let a, b be reals with $a < b$, and let $f : (a, b) \to \mathbb{R}$ be a function. Assume that f is differentiable on (a, b) and $f'(x) > 0$ for every $x \in (a, b)$. Prove that f is strictly increasing on (a, b).

2. *a*) Let f be a real-valued function on the closed interval $I = [a, b]$. (i) Define what a tagged partition is on I. (ii) Define what is meant by the width of a partition. (iii) Define what the Riemann sum is for f associated with a tagged partition of I.

b) Let f and I be as in Part *a*), and let A be a real number. Define what it means for A to be the Riemann integral of f on I.

3. *a*) Let $[a, b]$ be a closed interval and let $c \in [a, b]$ be real number. Define the function f on $[a, b]$ by

$$f(x) = \begin{cases} 1 & \text{if } x = c, \\ 0 & \text{otherwise,} \end{cases}$$

Using the definition of the Riemann integral, prove that $\int_a^b f = 0$.

b) State the Fundamental Theorem of Calculus. Make sure that the conditions are precisely stated. Do not give a proof.

4. *a*) Describe what a step function on an interval $[a, b]$ is.

b) State the second criterion (the one involving step functions) of Riemann integrability.

c) Let f be a real-valued function that is Riemann integrable on interval $[a, b]$. Prove that f is bounded on $[a, b]$.

5. *a*) State the necessary and sufficient condition for the series $\sum_{n=1}^{\infty} a_n$ of real numbers to be convergent (the condition in question is usually called the *Cauchy criterion* for convergence of series).

b) Prove that the harmonic series $\sum_{n=1}^{\infty} 1/n$ is divergent. Use material covered in the course (that is, *do not* use the Integral Test for convergence).