

Follow these instructions carefully:

Work on the paper provided; do not use your own paper. *Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet).* On the top of each page, *print* your name (*encircle your last name*) and indicate the number of the problem you are working on by writing e.g. “*Problem #4*”. Always *encircle* your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing “*Answer to Part b*)” (the number of the problem should be on the top of the page). Do not use a *red* pen or a *red* pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the *same* amount of credit. **Show all your work.**

1. Calculate the following limits:

a) $\lim_{x \rightarrow 0} (1 - 2x)^{1/x}$,

b) $\lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$.

2.a) Write the integral that finds the length of the curve $r = 3 - 2 \sin \theta$ in polar coordinates. *Do not calculate the integral.*

b) Write the integral that finds the area inside the curve $r = 4 \sin \theta$ and outside the curve $r = 3 - 2 \sin \theta$. *Do not calculate the integral.*

3. Decide whether the following improper integrals are convergent or divergent. Give clear reasons for your answer (no credit will be given for a correct answer unless the correct reason is also given). Do not calculate the integrals.

a) $\int_2^{+\infty} \frac{dx}{x^2 \ln x}$,

b) $\int_0^{+\infty} \frac{x dx}{x^2 + 1}$,

c) $\int_1^{+\infty} \frac{\ln x dx}{x^2 + 1}$.

4. Decide whether or not each of the following *sequences (not series)* is convergent. Give reasons for your answers. If the given sequence is convergent, find its limit.

a) $a_n = \frac{n-1}{n+1}$,

b) $a_n = \frac{2^n - 1}{2^n + 1}$,

c) $a_n = \cos n\pi$,

d) $a_n = \frac{\cos n\pi}{\sqrt{n}}$,

e) $a_n = \frac{\cos n}{n^2}$,

f) $a_n = (-1)^n \frac{n}{n+1}$.

5.a) Decide whether the series

$$\sum_{n=0}^{\infty} \frac{1}{\sqrt{n+1}}$$

is convergent.

b) Decide whether the series $\sum_{n=2}^{\infty} \frac{2^{3n-5}}{3^{2n-3}}$ is convergent. If it is convergent, find its sum.