Exam 3, Mathematics 1206, Section TY11 — 18 copies

11:00 am–1:00 pm, December 5, 2024, 1005-IH

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Follow these instructions carefully:

Work on the paper provided; do not use your own paper. *Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet).* On the top of each page, *print* your name (*encircle your last name*) and indicate the number of the problem you are working on by writing e.g. *"Problem #4"*. Always *encircle* your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing *"Answer to Part b)"* (the number of the problem should be on the top of the page). Do not use a *red* pen or a *red* pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the *same* amount of credit. **Show all your work.**

1. Decide whether each of the following series are convergent or divergent. Give reasons for your answers. No credit will be given for yes or no answers without proper explanations.

a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{4n}{3+9n}\right)^n$, *b)* $\displaystyle\sum_{n=1}^{\infty}\tan(1/n)$, *c)* $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n+n\sin^2 n}$,

d) $\displaystyle\sum_{n=1}^{\infty}\frac{n^2 2^n}{n!}$, *e)* $\displaystyle\sum_{n=1}^{\infty}\frac{2^n n!}{n^n}$.

2. How many terms of the series

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n!2^n}$$

need to be added so as to find the sum of this series with an error < 0.01? Give reasons for your answer, and show all your calculations needed to give the answer, but *do not* evaluate the sum.

3. Find the interval of convergence of the following power series. For each endpoint of the interval of convergence, decide whether the series is absolutely convergent, conditionally convergent, or divergent.

a) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n2^n}(x+3)^n$, *b)* $\displaystyle\sum_{n=1}^{\infty}\frac{(2x-12)^n}{8^n\sqrt{n}}$, *c)* $\displaystyle\sum_{n=1}^{\infty}n!x^n$.

4. Find the power series representation (centered at 0, i.e., in the form $\sum_{n=0}^{\infty}c_n x^n$) of

a) $\displaystyle\frac{1}{1+4x^2}$, *b)* $\displaystyle\frac{x}{(1+x)^2}$.

5. Find the first four terms of the Maclaurin series of $\dfrac{1}{\sqrt{1+x}}$ (that is, the last term you need to find will involve x^3). Make sure to write the coefficients as common fractions, *not as decimals*.