

1. (8 points) Suppose $h(x)$ has the following properties:

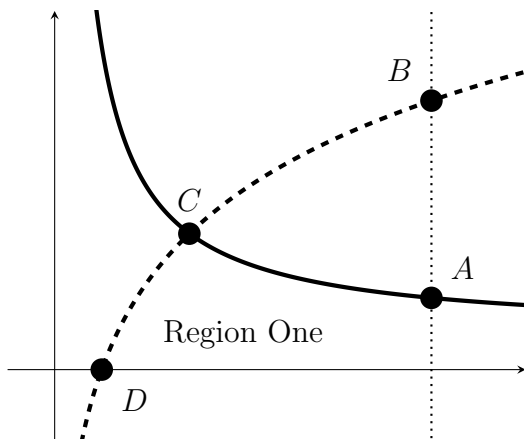
$$h(3) = 7$$

$$h(4) = 18$$

$$\int_3^4 e^{-2x} h(x) dx = 17$$

Find $\int_3^4 e^{-2x} h'(x) dx$

2. (6 points) The graphs of $x = 7$, $y = f(x)$ (solid) and $y = g(x)$ (dashed) are shown; the graph is **NOT** drawn to scale.



The points labeled are $A = (7, 3)$, $B = (7, 8)$, $C = (5, 5)$, and $D = (3, 0)$.

Write an expression to find the volume of the solid of revolution formed when Region One is revolved around the x -axis.

3. (10 points) Evaluate the following.

(a) $\int \frac{-24}{x^2 + 4x - 5} dx =$

(b) Suppose $f(x) = \frac{1}{7 - 5x} + \frac{3x + 5}{x^2 + 1}$. Find $\int f(x) dx$.

4. (9 points) Consider the integral $\int \frac{1}{\sqrt{9 - 25x^2}} dx$.

(a) To evaluate it using a trigonometric substitution, use the substitution

$x =$ _____

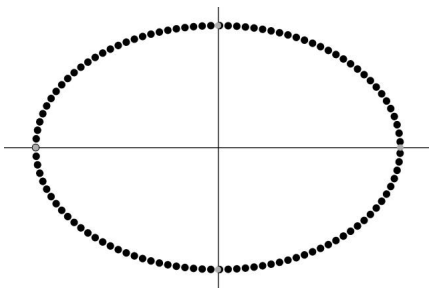
(b) This transforms the integral into

$\int \frac{1}{\sqrt{9 - 25x^2}} dx =$ _____ (**DO NOT** evaluate the integral).

(c) Integrate.

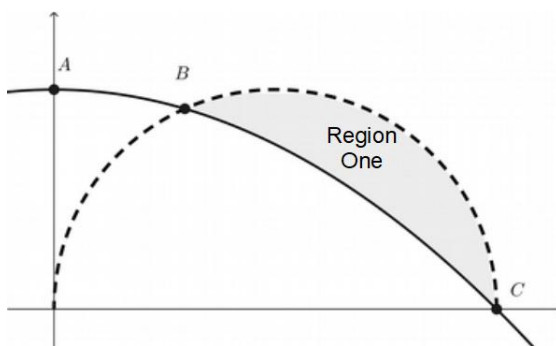
$\int \frac{1}{\sqrt{9 - 25x^2}} dx =$ _____

5. (6 points) A comet move along an elliptical orbit with with equation $25x^2 + y^2 = 225$ (graph shown below, **NOT** drawn to scale)



Write an expression giving the total length of the path traveled by the comet in one orbit. (DO NOT attempt to evaluate the expression)

6. (6 points) The graph shows $r = f(\theta)$ (solid) and $r = g(\theta)$ (dashed), both expressed and graphed in polar coordinates. The graph is **NOT** drawn to scale.



The points indicated are $A = \left[7, \frac{\pi}{2}\right]$, $B = \left[8, \frac{\pi}{8}\right]$, and $C = [10, 0]$. Write an expression to find the area of Region One.

7. (8 points) If the integral shown converges, give its value; if it diverges, show or explain why.

$$(a) \int_6^{10} \frac{1}{(5x - 40)^5} dx$$

$$(b) \int_0^{\infty} \frac{x}{\sqrt{1 + 25x^2}} dx$$

8. (4 points) Suppose you know the following:

- $f(-1) = 1$, $f'(-1) = 2$,
- $g(-1) = 5$, $g'(-1) = -10$,

Assume f and g are continuous and continuously differentiable. Find $\lim_{x \rightarrow -1} \frac{\ln(g(x) - 4)}{[f(x)]^2 - 1}$

9. (9 points) For a physics project, Catherine uses a function $f(x)$ that satisfies the following:

- $f(9) = 3$,
- $f'(x) = -4f(x)$ for all x ,
- $f(x)$ and all its derivatives exist at $x = 9$.

(a) The third degree Taylor series for f based at $x = 9$ is:

(b) Suppose Catherine wanted to evaluate $f(9.03)$ to within 0.001. To do so, she will need to at the first _____ terms of the Taylor series for f based at $x = 9$. (Enter the MINIMUM number of terms she will need)

(c) To within 0.001, $f(9.03) \approx$ _____

10. (8 points) Suppose $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{n!}$

(a) The first three terms of the Maclaurin series for $\int x^5 f(x) dx$ are

(b) The general form of the terms will be

11. (9 points) Suppose Jesse determines the Taylor series for f based at $x = 3$ is

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-3)^n}{n (5^n)}$$

(a) The series is absolutely convergent in the interval

(b) The series is conditionally convergent at

(c) Which of the following can Jesse find using the series? Circle all that apply. (You do **NOT** need to compute the values; just indicate whether they can be found using the series above)

- $f(-2)$
- $f(3)$
- $f(5)$
- $f(8)$

12. (10 points) Consider the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt[5]{\ln n}}$.

(a) This series is (circle ONE)

Absolutely
Convergent

Conditionally
Convergent

Divergent

Convergence Cannot
Be Determined

(b) A test that will **NOT** determine whether this series is absolutely convergent is (choose ALL that apply):

Integral
Test

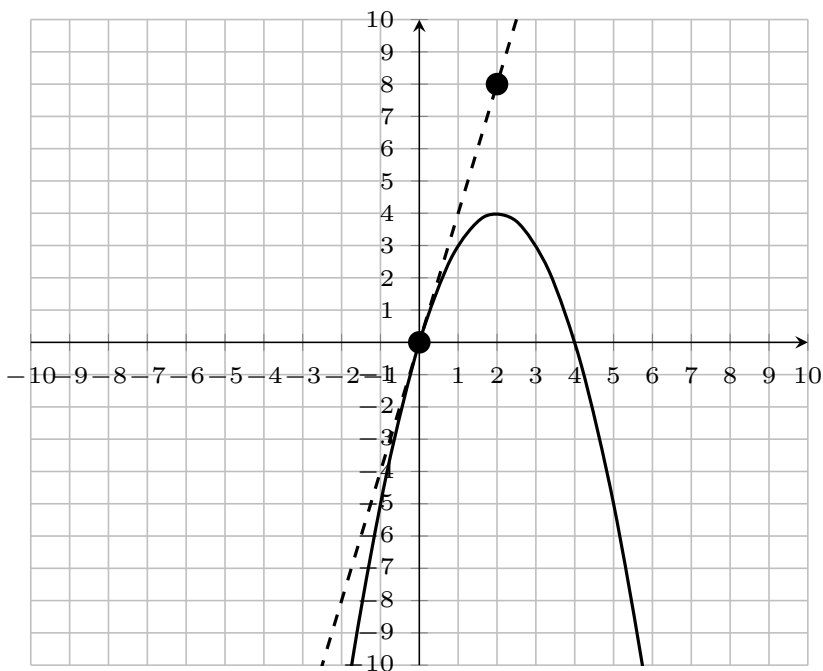
Root
Test

Ratio
Test

Limit Comparison
Test

(c) Show your work below. Show **BOTH** the results of a test that determines convergence or divergence, **AND** the results of a test that fails to determine convergence or divergence.

13. (4 points) The graph of $y = f(x)$ (solid) and its tangent line at $x = 0$ (dashed) is shown; a second point on the tangent line is also marked. Assume $f'(x)$ is continuous.



Find $\lim_{x \rightarrow 0} \frac{f(x)}{4 - 4e^x} =$

14. (3 points) Let $h(x) = \ln(8x + 8) \arctan(3x)$. Find $h'(x)$.