Exam 2, Mathematics 1206, Section TY9 — 13 copies

9:05 am–11:40 pm, October 26, 2023, 1005-IH

Instructor: Attila Máté

Follow these instructions carefully:

Work on the paper provided; do not use your own paper. *Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet).* On the top of each page, *print* your name (*encircle your last name*) and indicate the number of the problem you are working on by writing e.g. *"Problem #4"*. Always *encircle* your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing *"Answer to Part b)"* (the number of the problem should be on the top of the page). Do not use a *red* pen or a *red* pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the *same* amount of credit. **Show all your work.**

1. Calculate the following limits:

a) $\displaystyle\lim_{x\to+\infty} x(\ln(x+3)-\ln x)$, *b)* $\displaystyle\lim_{x\to+\infty} (xe^{1/x}-x)$.

2. Decide whether the following improper integrals are convergent or divergent. Give clear reasons for your answer (no credit will be given for a correct answer unless the correct reason is also given). Do not calculate the integrals.

a) $\displaystyle\int_2^{+\infty}\frac{dx}{(x+1)(\ln x)^2}$, *b)* $\displaystyle\int_0^{+\infty}\frac{dx}{\sqrt{x^2+1}}$, *c)* $\displaystyle\int_1^{+\infty}\frac{x\ln x\,dx}{x^3+1}$.

3. *a)* Decide whether the improper integral $\displaystyle\int_0^{1/2}\frac{dx}{x(\ln x)^2}$ is convergent. If it is convergent, calculate the integral.

b) Write the integral that finds the area that is the inside the curve $r=3\cos\theta$ (given in polar coordinates) and outside the curve $r=2-\cos\theta$. *Do not evaluate the integral!*

4. Decide whether or not each of the following sequences is convergent. Give reasons for your answers. If the given sequence is convergent, find its limit.

a) $a_n=\dfrac{n-2}{n+3}$, *b)* $a_n=\dfrac{3^n-n}{3^n+n^2}$, *c)* $a_n=\sin n\pi$,

d) $a_n=\dfrac{\ln n}{n}$, *e)* $a_n=\dfrac{\sin n}{n^2}$, *f)* $a_n=(-1)^n\dfrac{n-1}{n+1}$.

5. *a)* Decide whether the sum $\displaystyle\sum_{n=1}^{\infty}\frac{8^{n+1}}{9^n}$ is convergent. If it is convergent, find its limit.

b) Decide whether the sum $\displaystyle\sum_{n=3}^{\infty}\frac{1}{n(n+2)}$ is convergent. If it is convergent, find its limit.