Exam 3, Mathematics 1206, Section TY9 13 copies
9:05 am–11:40 pm, November 30, 2023, 1005-IH
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Follow these instructions carefully:

Work on the paper provided; do not use your own paper. *Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet).* On the top of each page, *print* your name (*encircle your last name*) and indicate the number of the problem you are working on by writing e.g. *"Problem #4"*. Always *encircle* your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing *"Answer to Part b)"* (the number of the problem should be on the top of the page). Do not use a *red* pen or a *red* pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the *same* amount of credit. **Show all your work.**

1. Decide whether each of the following series is convergent. Give reasons for your answers. No credit will be given for yes or no answers without proper explanations.

a) $\displaystyle\sum_{n=1}^{\infty}\frac{n\ln n}{n^2+1}$ *b)* $\displaystyle\sum_{n=2}^{\infty}\frac{n^2-1}{n^4+2}$ *c)* $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt[3]{n-1}}{\sqrt{n^3+1}}$

2. Decide whether each of the following series is absolutely convergent, conditionally convergent, or divergent. Give reasons for your answer. No credit will be given for a correct answer without a correct explanation.

a) $\displaystyle\sum_{n=2}^{\infty}(-1)^{n-1}\frac{\ln n}{n}$, *b)* $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n^2}{n^2+\ln n}$, *c)* $\displaystyle\sum_{n=2}^{\infty}(-1)^{n-1}\frac{n-1}{n^2(\ln n)^2}$.

3. Find the interval of convergence of the following power series. For each endpoint of the interval of convergence, decide whether the series is absolutely convergent, conditionally convergent, or divergent.

a) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n3^n}(x+4)^n$, *b)* $\displaystyle\sum_{n=1}^{\infty}\frac{(2x-7)^n}{5^n\sqrt{n}}$, *c)* $\displaystyle\sum_{n=1}^{\infty}n!(x-1)^n$.

4. *a)* Find the interval of convergence of the series

$$\sum_{n=1}^{\infty}\frac{x^n}{5^n n^2}.$$

Be sure to decide whether the series is absolutely convergent, conditionally convergent, or divergent at each endpoint of the interval of convergence.

b) Find the first four terms of the Taylor series of $1/x$ at $x=1$. That is, find the terms involving 1, $x-1$, $(x-1)^2$, $(x-1)^3$ in the Taylor expansion

$$\frac{1}{x}\sim\sum_{n=0}^{\infty}c_n(x-1)^n.$$

5. a) Use the Maclaurin series $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ to write the Maclaurin series of $\int_0^x e^{-t^2}\, dt$.

b) Using the results of part a) of this problem, find $\int_0^{1/4} e^{-x^2}\, dx$ with four decimal accuracy.