

[A significant portion](#) of the final exam will be based on the following archetypes (however, anything in the "instructor's guide" is fair game!) Again, at least 50% of the final exam will be "word problems," so prepare your students!

[Also note that the "Instructor's Guide" is your best guide to what you should be teaching; this document is updated "when and if."](#)

Integration Techniques

What we're up against:

<https://www.integral-calculator.com/>

Any problem where students are given a function to integrate can be done using an online calculator, which will also show all the steps necessary.

Question T1

Suppose $f(x)$ has the following properties:

- $f(5) = 25$ and $f(0) = 8$
- $\int_0^5 f(x) dx = 3$
- $\int_0^5 xf(x) dx = 10$

Evaluate: $\int_0^5 (x^2 + 3x + 7)f'(x) dx$

Variations: Given value of $\int_0^5 e^{-x} f(x) dx$, $\int_0^5 \frac{f(x)}{x} dx$, $\int_0^5 f(x) \sin x dx$, etc.

MyOpenMath notes

See 451571, 451600, 451878

Question T2

Rewrite (but do not evaluate) the following integral using integration by parts:

$$\int \frac{1}{(x+3)(9x^2+4)} dx$$

Multiple choice question, with answers:

- (a) integration by parts version 1 (using $u = 1/(x + 3)$)
- (b) integration by parts version 2 (using $u = 1/(9x^2 + 4)$)
- (c) wrong answer ($uv - \int u dv$, maybe)
- (d) wrong answer ($\int u dv - uv$)

(Reza will send some samples)

MyOpenMath notes

The linear factor should be of the form $x + a$, where a is a nonzero integer between -10 and 10.

The quadratic factor should be of the form $b^2 x^2 + c^2$, where b, c are integers from 2 to 5.

Question T3

Rewrite (but do not evaluate) the following integral using a trigonometric substitution:

$$\int \frac{1}{(x + 3)(9x^2 + 4)} dx$$

Multiple choice question, with answers:

- (a) $\tan \theta = 3x/2$
- (b) various errors.

(We have to be careful here: $x = \cos \theta$ is, technically, a trigonometric substitution.)

Question T3

To evaluate the integral $\int \frac{1}{\sqrt{x^2 + 4x + 5}} dx$ using a trigonometric substitution, use the substitution (type "theta" for θ).

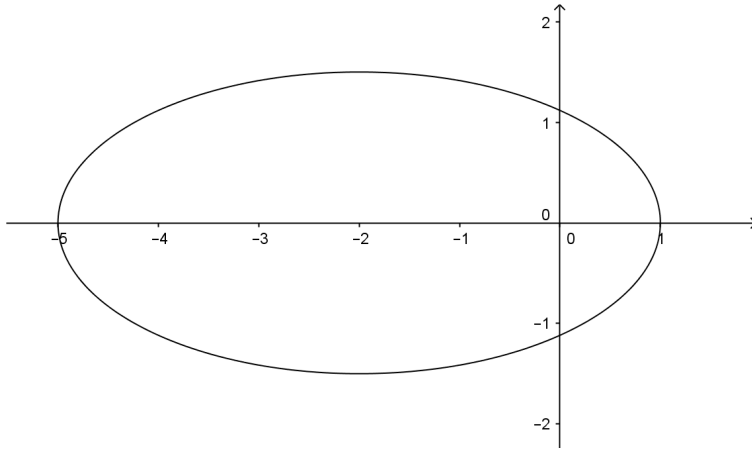
This transforms the integral into

$$\int \frac{1}{\sqrt{x^2 + 4x + 5}} dx = \int \text{sec}(\theta) d\theta$$

Applications of the Integral

Question A1

An asteroid moves in an elliptical orbit with equation $x^2 + 12x + 4y^2 + 20y = 400$ (the graph is shown below). Set up but **do not evaluate** an expression to determine the distance the asteroid travels in one orbit. (Assume the asteroid is moving clockwise along the ellipse)



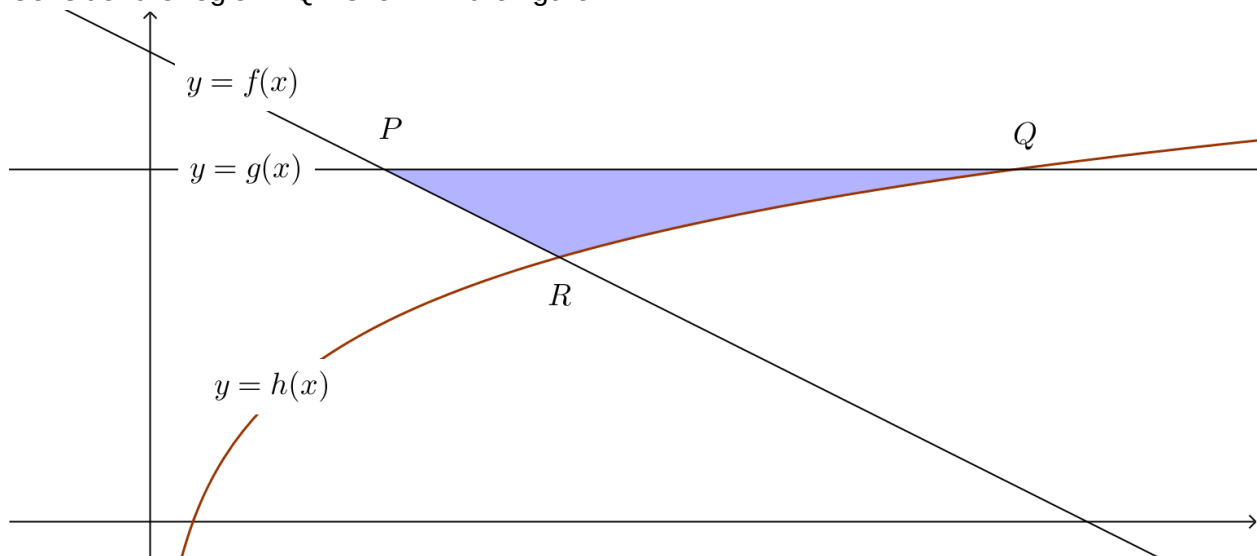
Make multiple choice, with different possibilities for answers.

[MyOpenMath Notes](#)

[Compare 450802.](#)

Question A3

Consider the region PQR shown in the figure.



Let $P = (1, 4)$, $R = (2, 2)$, and $Q = (8, 4)$.

Suppose this region is to be revolved around the x-axis. to produce a solid of revolution.

Set up two ways, don't integrate.

You have learned three different methods of using integration to find volumes of solids of revolution: shells, disks, and washers.

Which of these methods is **not** suitable for computing the volume of the solid of revolution? Explain why.

Write down integral expressions corresponding to computing the volumes using **each** of the other two methods.

Suppose $f(x)$ and $g(x)$ are linear functions, but $h(x) = \ln(ax + b)$ for some values a, b (**DO NOT** attempt to find a, b : their values are irrelevant for this question). Which of your two integral expressions would be easier to evaluate? Why?

[MyOpenMath notes](#)

Multiple choice versions: [452631](#), [452640](#), [452641](#), [452649](#)

L'Hopital's Rule

Question H1

Suppose $f(x)$ and $g(x)$ are continuous with continuous derivatives, with the following values:

x	3	4	5
$f(x)$	8	4	1
$f'(x)$	2	6	13
$g(x)$	5	1	4
$g'(x)$	-3	-8	-4

Find the following.

$$\lim_{x \rightarrow 4} \frac{[f(x)]^2 - 16}{\ln g(x)}$$

$$\lim_{x \rightarrow 4} (g(x) - 1)^{16 - [f(x)]^2}$$

[MyOpenMath notes](#)

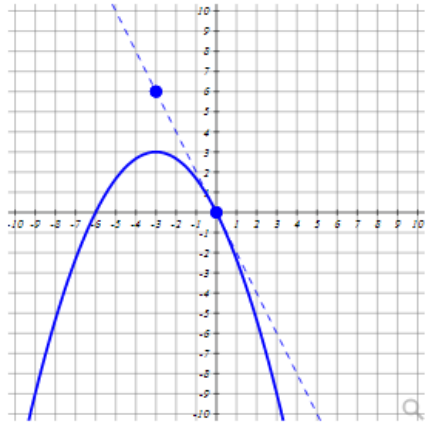
See [417775](#), [453329](#), [453348](#), [453392](#), [464159](#)

(Also include some more "standard" computational questions)

Question H2

MyOpenMath - Test Question - Google Chrome
myopenmath.com/course/testquestion2.php?cid=77696&qsetid=453380

The graph of $y = f(x)$ (solid) and its tangent line at $x = 0$ (dashed) is shown; a second point on the tangent line is also marked. Assume $f'(x)$ is continuous.



Find

$$\lim_{x \rightarrow 0} \frac{f(x)}{x} = \text{[input box]} \text{ } \sigma^6$$
$$\lim_{x \rightarrow 0} \frac{f(x)}{3e^x - 3} = \text{[input box]} \text{ } \sigma^6$$

Show Detailed Solution
Question Help: [Written Example](#)

Submit New Version

[MyOpenMath notes](#)

See 453380

Question H3

Suppose

$$f(2) = -2$$

$$f'(2) = 5$$

$$g(2) = -5$$

$$g'(2) = 7$$

$$\lim_{x \rightarrow 2} \frac{(f(x) + 2)^3}{(g(x) + 5)^2} =$$

Cannot Be Determined



[MyOpenMath Notes](#)

See 463311, 463312, 463314

[Question H4](#)

Suppose $f(x)$ and $g(x)$ have the following values:

- $\lim_{x \rightarrow \infty} f(x) = \infty$ and $\lim_{x \rightarrow \infty} f'(x) = -10$,
- $\lim_{x \rightarrow \infty} g(x) = \infty$ and $\lim_{x \rightarrow \infty} g'(x) = 1$.

Assume all functions are continuous. Find the following.

$$\lim_{x \rightarrow \infty} \frac{[f(x)]^2 - 3f(x) + 2}{[g(x)]^2 - 1} =$$

Question Help: [Video](#)

Improper Integrals

Question M2

Assume $f(x)$ is continuous and positive everywhere, and that $\int_1^{\infty} f(x) dx$ converges. Find the following.

a) Assuming it exists, $\lim_{x \rightarrow \infty} f(x)$

b) Assuming it exists, $\lim_{x \rightarrow \infty} f'(x)$

Decide on the convergence or divergence of the following ("cannot be determined" is also an option). Defend your conclusion.

$$\int_{10}^{\infty} f(x) dx$$

$$\int_1^{\infty} f(x + 3) dx$$

$$\int_1^{\infty} (f(x) + 3x) dx$$

$$\int_1^{\infty} \frac{f(x)}{x} dx$$

$$\int_1^{\infty} f(x^2) dx$$

$$\int_1^{\infty} f(x) \sin x dx$$

MyOpenMath notes

See 456805, 452658.

For the Instructor

Except for the first few, this is actually a bunch of "integral test" questions. The way to approach

these problems is through the integral test: the convergence of $\int_1^{\infty} f(x) dx$ means that the series with terms $f(n)$ converges. Given that information, the convergence or divergence of the others can be determined, which determines whether the integral converges.

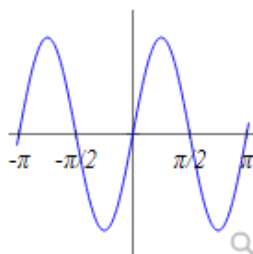
Polar Coordinates

Question R1

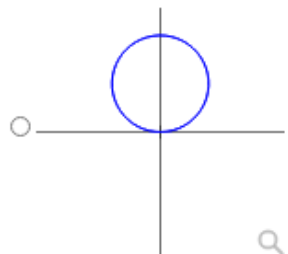
MyOpenMath - Test Question - Google Chrome

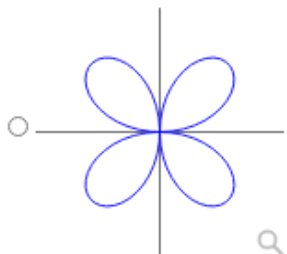
myopenmath.com/course/testquestion2.php?cid=77696&qsetid=456824

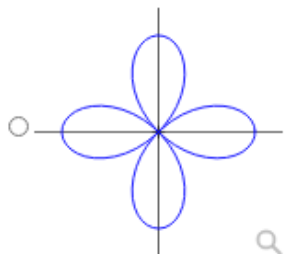
Jamie tried to produce the polar graph $r = f(\theta)$, but instead produced the rectangular graph of $y = f(x)$



Which of the following is the graph she should have produced?







Question Help: [Video](#)

[MyOpenMath notes](#)

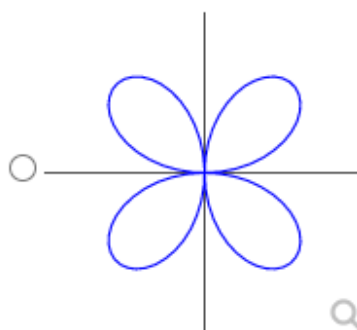
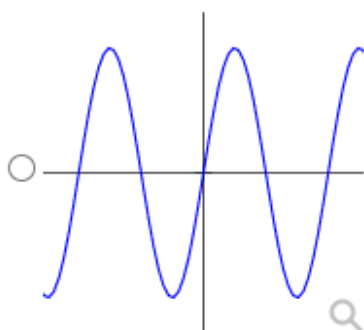
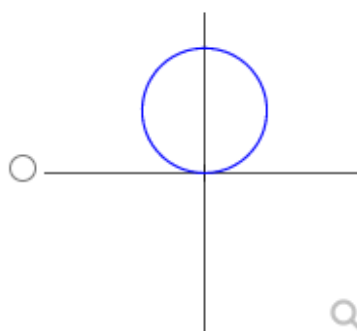
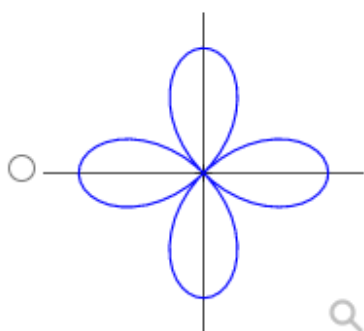
See 456821, 456824.

Question R3

Prev Next 0 checked 0 remaining

 Mark Question for Use

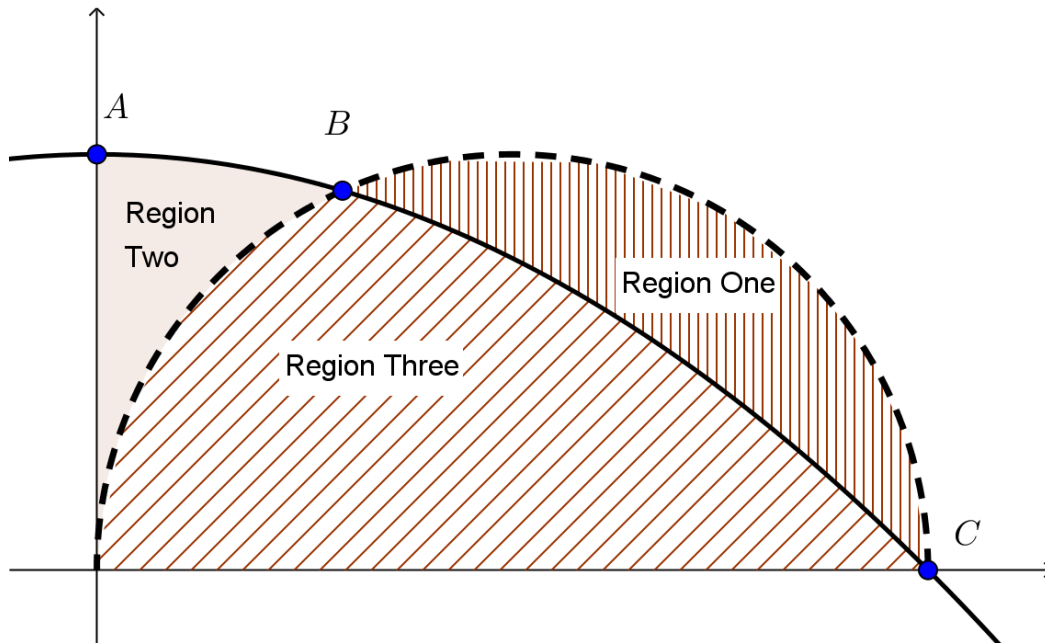
Supposes $a > 0$. Which of the following might represent the polar graph $r = a \sin 2\theta$?

Question Help: [Video](#)Question ID: 456815. Seed: 2289. [Edit Question](#)Description: Graphing in Polar Coordinates (choose graph of $r = a \sin/\cos b\theta$, a positive, internet resistant)[MyOpenMath notes](#)

See 456807, 456813, 456815

Question R2

The graph shows $r = f(\theta)$ (solid) and $r = g(\theta)$ (dashed); the points have coordinates $A = [4, \pi/2]$, $B = [3, \pi/3]$, $C = [4, 0]$. Write an expression for the area of Region One; Region Two; and Region Three.



Other possibilities:

MyOpenMath notes

See 454478, 454509, 454513 (multiple choice, select integrals)

These can use generic graphs; the intersection point can be any point in the first quadrant (and we don't even have to use multiples of π).

Series Convergence

Symbolab can answer series convergence questions, including determining the appropriate test and showing all the steps necessary to solve the problem.

Question S1

(This one may be too hard for students, and too hard to grade, but it might be a good question to ask during the semester as formative assessment, possibly with easier series?)

During the semester, you have learned or utilized the following tests for series convergence: nth term test; ratio test; root test; limit comparison test; integral test.

Consider the series $\sum \frac{(2n + 5)^3}{\sqrt{n^8 + 5n + 23}}$.

Of the five series convergence tests, identify two tests which will be inconclusive and will not provide information about whether the series converges or diverges; show your work.

The _____ test is inconclusive, because (show work below).

Additionally, the _____ test is inconclusive, because (show work below).

Of the remaining tests, identify two tests that **will** conclusively determine the convergence or divergence of the series. Explain why you reject one, then set up the other and decide whether or not the series converges.

The _____ test will conclusively determine the convergence of the series. However, we don't want to use it because

The _____ test will also conclusively determine the convergence of the series.

We'll use the _____ test to determine the convergence by evaluating

[MyOpenMath notes](#)

[Compare 397150, 397135 \(for identifying "no answer" tests\)](#)

Question S2

(Give series, then choice of tests, and indicate whether test indicates convergence/divergence/inconclusive/test cannot be applied)

Example: $1/n$: integral, limit test, etc.

Power Series

Question P1

Suppose $f(x)$ has the following properties:

- $f(x)$ and all its derivatives exist at $x = 5$.
- $f(5) = 8$.

- $f'(x) = \frac{1}{[f(x)]^2}$ for all x .

MyOpenMath notes

See 454516, 456833

Question P3

$$f(x) = \sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n (2x)^n$$

Suppose

- Find a series expression for $\int x^3 f(x) dx$
- If possible, use your series to approximate $\int_0^2 x^3 f(x) dx$. If not possible, explain why not.

MyOpenMath notes

Compare 456849, 456845, 456852

Question P4

MyOpenMath - Test Question - Google Chrome

myopenmath.com/course/testquestion2.php?cid=77696&qsetid=4...

Prev Next 0 checked 2 remaining

Mark Question for Use

Suppose

$$f(x) = \sum_{n=0}^{\infty} \frac{2^n}{n!} (x - 7)^n$$

To determine $f(6.9)$ to within 0.0001, it will be necessary to add the first terms of the series.

$f(6.9) \approx$ (Enter the answer accurate to four decimal places)

[Show Detailed Solution](#)

Question Help: [Video](#) [Written Example](#)

[MyOpenMath notes](#)

See 456841