Exam 2, Mathematics 2501, Section TR2 — 24 copies

2:15–3:25 pm, March 26, 2026, IH-1105

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Follow these instructions carefully:

Work on the paper provided; do not use your own paper. *Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet).* On the top of each page, *print* your name (*encircle your last name*) and indicate the number of the problem you are working on by writing e.g. *"Problem #4"*. Always *encircle* your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing *"Answer to Part b)"* (the number of the problem should be on the top of the page). Do not use a *red* pen or a *red* pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the *same* amount of credit. **Show all your work.**

1. Let X be the random variable with density function

$$f_X(x) = \begin{cases} 4x^{-5} & \text{if } x \ge 1, \\ 0 & \text{otherwise.} \end{cases}$$

a) Find the expectation of X.
b) Find the variance of X.
c) Find the expectation of X^3.

2. In a factory, certain items are produced. Each item turns out to be defective with a probability of .001, independently of what happens to other items. After production, without checking the items, they are packed in boxes of 1600 each.

a) Assume a large number items are produced. Write an exact formula for the probability that a box contains exactly k defective items ($0 \le k \le 1600$).

b) Using an appropriate approximation, calculate the probability that a box contains 4 defective items.

3. A lifetime X of a certain type of light bulb is described by the exponential distribution with density function $f_X(x) = \theta^{-1} e^{-x/\theta}$ ($x \ge 0$); then θ is the mean lifetime. A new bulb is switched on, and it is found that its lifetime is 2000 hours. Construct a two-sided confidence interval for θ with confidence level 90%. Hint: the distribution function of X is

$$F_X(x) = P(X \le x) = \int_{-\infty}^{x} f_X(t)\,dt = \int_0^x \frac{1}{\theta} e^{-t/\theta}\,dt = 1 - e^{-x/\theta}.$$

4. The following four measurements of the distance between two points are taken:

42.1, 45.3, 37.9, 41.5.

This is regarded as a random sample from a distribution $\mathcal{N}(m, \sigma^2)$, where m is the true distance, and σ^2 measures the precision of the method.

a) Find an unbiased estimate for m.
b) Find an unbiased estimate for σ^2 if it is known that $m = 40.0$.
c) Find an unbiased estimate for σ^2 if m is unknown.

5. The following four measurements of the distance between two points are taken:

42.1, 45.3, 37.9, 41.5.

This is regarded as a random sample from a distribution $\mathcal{N}(m, 9)$, where m is the true distance, and $\sigma^2 = 9$ measures the precision of the method; the value of m is not known. Find a 95% confidence interval for m.