Exam 2, Mathematics 2501, Section TR3 21 copies
3:40–4:50 pm, March 26, 2026, IH-1105
Instructor: Attila Máté

Follow these instructions carefully:

Work on the paper provided; do not use your own paper. *Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet).* On the top of each page, *print* your name (*encircle your last name*) and indicate the number of the problem you are working on by writing e.g. *"Problem #4".* Always *encircle* your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing *"Answer to Part b)"* (the number of the problem should be on the top of the page). Do not use a *red* pen or a *red* pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the *same* amount of credit. **Show all your work.**

1. Let p be a real with $0 < p < 1$ and let $q = 1 - p$. Let U be a discrete random variable such that $\mathrm{P}(U = 1) = p$ and $\mathrm{P}(U = 0) = q$. In all parts of this problem, you must show and explain your calculations. Giving only the final answers will get zero credit even if the answers are correct.

a) Find the expectation of U.
b) Find the variance of U.
c) Using the result in part *a*), find the expectation of a binomial variable $\mathrm{Bin}(n, p)$.
d) Using the result in part *b*), find the variance of a binomial variable $\mathrm{Bin}(n, p)$.

2. A fair die is tossed 450 times. Let X denote the number of times that a number ≤ 4 is obtained.

a) Write a formula for the probability $\mathrm{P}(X = k)$ $(0 \leq k \leq 450)$.
b) Write the sum describing the probability $\mathrm{P}(X \leq 305)$
c) Find a numerical approximation for the probability $\mathrm{P}(X \leq 305)$; make sure to write the formula you are using to obtain the approximation.

3. The following four measurements of the distance between two points are taken:

$$42.1, \qquad 45.3, \qquad 37.9, \qquad 41.5.$$

This is regarded as a random sample from a distribution $\mathcal{N}(m, \sigma^2)$, where m is the true distance, and σ^2 measures the precision of the method.

a) Find an unbiased estimate for m.
b) Find an unbiased estimate for σ^2 if it is known that $m = 40.0$.
c) Find an unbiased estimate for σ^2 if m is unknown.

4. The following for measurements of the distance between two points are taken:

$$42.1, \qquad 45.3, \qquad 37.9, \qquad 41.5.$$

This is regarded as a random sample from a distribution $\mathcal{N}(m, \sigma^2)$, where m is the true distance, and σ^2 measures the precision of the method; the values of m and σ^2 are not known. Find a 95% confidence interval for m.

5. The following for measurements of the distance between two points are taken:

$$42.1, \qquad 45.3, \qquad 37.9, \qquad 41.5.$$

This is regarded as a random sample from a distribution $\mathcal{N}(m, \sigma^2)$, where m is the true distance, and σ^2 measures the precision of the method; the values of m and σ^2 are not known.

a) Find a 95% confidence interval for σ^2.
b) Find a 95% confidence interval for σ.