Exam 3, Mathematics 2501, Section TR2 — 24 copies

2:15–3:25 pm, May 7, 2026, IH-1105

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Follow these instructions carefully:

Work on the paper provided; do not use your own paper. *Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet).* On the top of each page, *print* your name (*encircle your last name*) and indicate the number of the problem you are working on by writing e.g. *"Problem #4"*. Always *encircle* your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing *"Answer to Part b)"* (the number of the problem should be on the top of the page). Do not use a *red* pen or a *red* pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the *same* amount of credit. **Show all your work.**

1. Consider a random sample of a normal distribution with standard deviation 2 and unknown expectation μ (i.e., of the distribution $\mathcal{N}(\mu, 4)$):

$$46.1, \qquad 50.4, \qquad 43.5, \qquad 44.0.$$

The hypothesis

$$H_0 : \mu = 48.0$$

is to be tested against the alternative hypothesis

$$H_1 : \mu > 48.0$$

(one-sided test).

a) Find the p-value of this test.

b) Test H_0 at the level 5% of significance.

c) Test H_0 at the level 1% of significance.

2. Consider a random sample of a normal distribution with unknown standard deviation and unknown expectation μ (i.e., of the distribution $\mathcal{N}(\mu, \sigma^2)$ with unknown σ):

$$46.1, \qquad 50.4, \qquad 43.5, \qquad 44.0.$$

The hypothesis

$$H_0 : \mu = 48.0$$

is to be tested against the alternative hypothesis

$$H_1 : \mu > 48.0$$

a) Test H_0 at the level 5% of significance.

b) Explain why it is not feasible to find the p-value of this test without a computer.

3. In a random sample of 600 persons eating lunch at a department store cafeteria, 360 persons had dessert. Construct a 90% confidence interval for the true proportion of persons usually eating dessert in this cafeteria.

4. *a*) Let P, Q, and R be events, and let I_P, I_Q, and I_R be their indicator variables. Describe the event whose probability equals the expectation

$$\mathrm{E}\big(1 - (1 - I_P)(1 - I_Q)(1 - I_R)\big)$$

b) Given events P, Q, and R, use indicator variables to derive the formula for $\mathrm{P}(P \cup Q \cup R)$.

5. Let $n > 0$ be an integer,and $X \sim \mathrm{Bin}(n, x)$ be a binomial variable. Given $\delta > 0$, show that

$$\sum_{\substack{k:0\le k\le n\\ |k-xn|\ge n\delta}} \binom{n}{k} x^k (1-x)^{n-k} \le \frac{1}{n\delta^2}.$$